KINEMATICS OF WAVES INTERACTIONS IN UNDULAR FRAMES

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Abstract

In this paper the concept of undular frame is defined as a system of tools consisting of waves, having the same nature as observed waves. Such definition of used tools set is equivalent to the problem about behavior and self-organizing of waves in absence of heterogeneous objects. The theorem has proved that the velocity of signal propagation in medium does not depend on undular frame selected. The location of undular frames cannot be determined relatively to the medium-carrier of waves. Between undular frames the principle of relativity is completely observed, is not possible to distinguish any undular frame. If the waves interactions are considered in undular frames, then there are gained not only trivial solutions corresponding to a principle of superposition, but also solutions, which describe interactions between waves as between mechanical particles. At the interaction of stable standing wave with the traveling wave the quantization of the latter takes place.

1. Introduction

The undular processes are applied as the etalons of time and length as it is known. For example, as the time etalon, the period of oscillations of caesium atoms is chosen, and the wave length of krypton atoms radiation serve as length etalon. In other words, the measuring of time and lengths represents operations of comparison with the waves parameters: with the period T and wave length λ.

Usually it is considered, the etalons do not vary during measuring and they are heterogeneous in relation to investigated processes. In practice, any tool is exposed to action during measuring. The changes in tools are taken into account through errors. However there are cases, when the tools of measuring are the participants of processes. Hence, the changes, which happen to tools, cannot be taken into account with the help of errors. This situation takes place, when all objects, which participate during the measuring process, have an undular nature.

The subject of the present work is the study of wave interaction provided that as tools of time and length measuring serve waves, which exist in same medium, as investigated waves. Or else, we want to describe, how the waves “perceive” each other. Such statement, in fact, is equivalent to a problem about behaviour and self-organizing of waves in unbounded medium, without heterogeneous insertions. We have published separate results on examination of this problem in works [1-7]. In offered paper we systematized results on the kinematics of waves, described in undular frames.

2. The metric of undular frames

2.1. The definition of undular frames

The result of interaction of free physical object is its transition from one reference system in other. Therefore, first of all, we shall spot concept of the undular reference system, or frame, and the rules of transition from one system in another.

The reference system should contain scales for measuring the time and the length. Such scales consist of repeated intervals of time and length. The standing wave possesses the property of periodicity in space and in time, it can be described by expression:

$$a = A\cos(-kx)\cos(\omega t). \tag{1}$$

By setting such wave, we thus set the metrics, namely:

- the direction of an axis x - coincides with a direction of the wave propagation;
- the spatial gauge - is determined by the wave length $\lambda = \frac{2\pi}{k}$;
- the time gauge - is determined by the wave period $T = \frac{2\pi}{\omega}$. Or else, the standing wave (1) execute a role of the ruler and chronometer simultaneously.

If in medium there is a certain wave-object, described by expression:

$$a_0 = A\cos(-k_0 x)\cos(\omega_0 t), \tag{2}$$

the measuring of its length in a frame (1) consists in definition of the number equal to ratio of wave-object and wave-gauge lengths:

$$n = \lambda_0 / \lambda . \tag{3}$$

Similarly, the measurement of wave-object period consists in determination of ratio of wave-object and wave-gauge periods:

$$n = T_0 / T \tag{4}$$

The wave-object (2) can be decomposed into two travelling waves, which run in opposite directions, of a kind:

$$a_{01} = \frac{A}{2}\cos(\omega_0 t - k_0 x) \tag{5}$$

$$a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x). \tag{6}$$

Generally, in expressions (5) and (6) the frequencies and the wave numbers can differ, that is, these expressions take view:

$$a_{01} = \frac{A}{2}\cos(\omega_{01} t - k_{01} x) \text{ And } a_{02} = \frac{A}{2}\cos(\omega_0 t + k_0 x),$$

at that $\omega_{01} \neq \omega_0$ and $k_{01} \neq k_0$. Then the wave-object will be described by the formula:

$$a_0 = a_{01} + a_{02} = A_0 \cos\left(\frac{\omega_{01} - \omega_0}{2} t - \frac{k_0 + k_{01}}{2} x\right)\cos\left(\frac{\omega_{01} + \omega_0}{2} t - \frac{k_0 - k_{01}}{2} x\right). \tag{7}$$

Formula (7) describes so-called beats, or standing wave, the maximums of which are moving in the course of time. We shall term such wave - quasi-standing wave. At that, the value

$$\omega' = \frac{\omega_{01} + \omega_0}{2} \tag{8}$$

and

$$k' = \frac{k_{01} + k_0}{2} \tag{9}$$

are perceived as frequency and wave number of moving wave-object (7), and its velocity is determined by expression

$$v_0 = \frac{\Delta x}{\Delta t} = \frac{\omega_{01} - \omega_0}{k_0 + k_{01}}. \tag{10}$$

If the observer goes with velocity defined by expression (10), from his point of view the wave (7) will be standing, or fixed, and it will be described by expression having view (2), or, at n=1, by expression of view (1). Hence, in system of the moving observer this wave can be used as a wave, which sets a frame. Thus, we come to the conclusion, that within the framework of the accepted model, there can be many reference systems, which are moving one, relatively another with various velocities, and all of them are equivalent.

Definition: the undular reference system, or undular frame is a frame, in which as time etalon there serves the period of standing or quasi-standing wave in a fixed point, and the length etalon is equal to distance between two points having same phase.

2.2.Theorem on invariance of velocity of traveling wave relatively to undular frames.

As it is marked above, a preferred frame cannot exist. Or else, for undular frames the principle of relativity is realized. However there is one circumstance, which can put under doubt last statement. The velocity of traveling waves c, described by expressions (5) and (6) is determined by properties of medium. Naturally there can be the idea: to use a standing wave as the tool for determining velocity of traveling waves c. Then, knowing velocity c, to determine velocity of undular frame concerning the medium. In fact, such experience is similar to a known experiment of Michelson and Morley, with help of which the attempt was undertaken to spot velocity of the motion concerning the ether in 1887. In our case, if such experience will give positive result, then it will be possible to choose one "true undular frame", in which the velocity of the motion concerning the medium - carrier is equal to zero. Such system will be privileged in relation to other undular frames. In this case, for undular systems the principle of relativity will not be realized. Let's prove, that it not true.

Theorem: the velocity of traveling waves c has the same value in all undular frames.

We suppose there are two undular frames, which are described by expressions:

$$a = A\cos(-kx)\cos(\omega t), \qquad (11)$$

and

$$a' = A\cos\left(\frac{\omega_1 - \omega}{2}t - \frac{k + k_1}{2}x\right)\cos\left(\frac{\omega_1 + \omega}{2}t - \frac{k - k_1}{2}x\right). \qquad (12)$$

The relative motion velocity of these systems is determined by expression:

$$v = \frac{\omega_1 - \omega}{k + k_1} = c^2\frac{T - T_1}{\lambda_1 + \lambda}. \qquad (13)$$

Let admit that, some wave-object rest in system (12) and in this system it is described by the formula:

$$a_0' = A_0\cos(-k_0'x')\cos(\omega_0't'). \qquad (14)$$

The same wave-object, in system (11) will be described by expression

$$a_0 = A_0\cos\left(\frac{\omega_{01} - \omega_0}{2}t - \frac{k_0 + k_{01}}{2}x\right)\times\cos\left(\frac{\omega_{01} + \omega_0}{2}t - \frac{k_0 - k_{01}}{2}x\right). \qquad (15)$$

Let's copy (15) taking into account (13):

$$a_0 = A_0\cos\left(\frac{k_0 + k_{01}}{2}(vt - x)\right)\cos\left(\frac{\omega_0 + \omega_{01}}{2}(t - \frac{v}{c^2}x)\right). \qquad (16)$$

Expressions (16) and (14) describe the same wave-object. In expression (16) the value $(vt - x)$ represents instantaneous coordinate of wave-object, as well as x' in expression (14). The transformation of segment length along coordinate should take place under the same law as transformation of this coordinate. Hence, the length of moving wave-object (16) will be

$$\lambda_0' = \lambda_0 - vT_0,$$

and its wave number:

$$k_0' = \frac{2\pi}{\lambda_0 - vT_0}. \tag{17}$$

Following a similar reasoning for frequency, we shall obtain:

$$\omega_0' = \frac{2\pi}{T_0 - \frac{v}{c^2}\lambda_0}. \tag{18}$$

The ratio of circular frequency ω to a wave number k is equal to velocity of traveling wave c. Thus, the proof of the theorem formulated above is reduced to the demonstration of relation

$$c = \omega_0 / k_0 = \omega_0' / k_0' = c'.$$

By using (17) and (18) we shall obtain:

$$c' = \frac{\omega_0'}{k_0'} = c^2 \frac{\lambda_0 - vT_0}{c^2 T_0 - v\lambda_0}.$$

In view of expressions (3), (4) and (13) we can write:

$$c' = \frac{\lambda - Tc^2\left(\frac{T - T_1}{\lambda + \lambda_1}\right)}{T - \lambda\left(\frac{T - T_1}{\lambda + \lambda_1}\right)} = \frac{\lambda(\lambda + \lambda_1) - Tc^2(T - T_1)}{T(\lambda + \lambda_1) - \lambda(T - T_1)} = \frac{\lambda}{T} = c\ .$$

We have proved: in undular frames, the velocity of traveling wave c does not depend on choice of frame. Hence, the velocity of traveling wave cannot be used for definition of velocity concerning the medium-carrier, and all undular reference frames are equivalent.

2.3. Transformation of the length and time scales at transition from one undular system in another

We will found the transformation rules for segments of length and time intervals at transition from one undular system in another. Let's choose two undular frames. For distinguishing them, one of them we shall mark by an accent. We assume, that, the marked frame moves relatively not marked system with velocity v.

The measuring of the length segment and time interval in a undular frame consists in its comparison with the respective gauges, as which serve: the wave length λ and period T. Hence, for the segment of length x and time interval t in not marked system it is possible to note:

$$x = n\lambda, \quad t = nT\ ,$$

and for marked system:

$$x' = n\lambda', \quad t' = nT'. \tag{19}$$

The wave-object, situated in marked system, will be described by expression:

$$a_0' = A_0 \cos(-k_0' x')\cos(\omega_0' t'). \tag{20}$$

The same wave in not marked system will look like:

$$a_0 = A_0 \cos\left(\frac{\omega_{01} - \omega_0}{2}t - \frac{k_0 + k_{01}}{2}x\right) \times \cos\left(\frac{\omega_{01} + \omega_0}{2}t - \frac{k_0 - k_{01}}{2}x\right). \tag{21}$$

We shall term *proper frame* the reference frame, in which wave-object is situated. Comparing expressions (2) and (7), we can define a proper frame and as system, in which $\omega_1=\omega_2$ and $k_1=k_2$. According to expression (13), the velocity of proper system relatively to laboratory frame can be expressed as

$$v=\frac{c(\omega_{01}-\omega_0)}{\omega_{01}+\omega_0}=\frac{c(k_{01}-k_0)}{k_0+k_{01}}. \quad (22)$$

Relations from here follow:

$$k_{01}=k_0\frac{c+v}{c-v} \quad (23)$$

and

$$\omega_{01}=\omega_0\frac{c+v}{c-v} \quad . \quad (24)$$

By inserting (23) and (24) in (21) we shall obtain:

$$a_0=A_0\cos\left[\frac{k_0c}{c-v}(vt-x)\right]\cos\left[\frac{\omega_0c}{c-v}\left(t-\frac{v}{c^2}x\right)\right]. \quad (25)$$

Let's compare (20) and (25). These expressions describe the same wave-object. In both expressions, the arguments of first *cos* represent the same ratio of length segment to gauge i.e. to wave-object length, in respective reference frame. This is a dimensionless value, or simply a number, which does not depend on a reference frame. Therefore we can equate the arguments of *cos* from (20) and (25), and obtain:

$$x'=\frac{k_0}{k'_0}\frac{c}{c-v}(x-vt). \quad (26)$$

In correspondence with expression (17) and (19)

$$k_0'=\frac{2\pi}{\lambda_0-vT_0}=\frac{2\pi n}{x-vt}, \quad (27)$$

here n is number of waves lengths λ, located on a considered segment x.

Inasmuch all undular frames are equivalent, similarly, by transferring from marked system in the not marked system, it is possible to obtain the expression:

$$k_0=\frac{2\pi}{\lambda_0'-v'T_0'}=\frac{2\pi n}{x'-v't'}. \quad (28)$$

In this expression v'- represent the velocity, with which not marked frame move from "point of view" of marked system. Hence:

$$v'=-v,$$

and (28) will be copied:

$$k_0=\frac{2\pi n}{x'+vt'}. \quad (29)$$

Let's insert (27) and (29) in (26):

$$x'=\frac{\dfrac{2\pi n}{x'+vt'}}{\dfrac{2\pi n}{x-vt}}\frac{c}{c-v}(x-vt)=\frac{c}{(c-v)(x'+vt')}(x-vt)^2.$$

In correspondence with (19)

$$x'/t'=n\lambda'/nT'=c.$$

Hence:

$$x'=\frac{1}{t'}\frac{c}{(c-v)(c+v)}(x-vt)^2.$$

That is equivalent to:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x - vt}{\sqrt{1 - \beta^2}} = \gamma(x - vt), \qquad (30)$$

where $\beta = \frac{v}{c}$ - is- normalized velocity,

and

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad . \qquad (31)$$

We remind, in our example the proper system is the marked system. Let the length of segment in the proper system be equal to:

$$\Delta x' = x_2' - x_1' .$$

We designate as Δx the length of segment in system, relatively which it moves. Then, in correspondence with (30), we shall obtain:

$$\Delta x' = x'_2 - x'_1 = \gamma(x_2 - x_1) = \gamma \Delta x . \qquad (32)$$

Having done a similar transformations with argument of second *cos* in (25), we shall obtain the expression for proper time, corresponding to (26):

$$t' = \frac{\omega_0}{\omega_0'} \frac{c}{c - v}(t - \frac{v}{c^2} x) . \qquad (33)$$

Similarly to formulas (27) and (28), for frequencies it is possible to write the expressions:

$$\omega_0' = \frac{2\pi}{T - \frac{v}{c^2}\lambda} = \frac{n2\pi}{t - \frac{v}{c^2}x} , \qquad (34)$$

$$\omega_0 = \frac{2\pi}{T' - \frac{v'}{c^2}\lambda'} = \frac{n2\pi}{t' + \frac{v}{c^2}x'} . \qquad (35)$$

Here, n is the number of wave-object periods T, elapsed from the beginning of readout up to a considered instant. We insert (34) and (35) in (33) and obtain:

$$t' = \frac{\dfrac{n2\pi}{t' + \frac{v}{c^2}x'}}{\dfrac{n2\pi}{t - \frac{v}{c^2}x}} \frac{c}{c - v}(t - \frac{v}{c^2}x) = \frac{c}{(t' + \frac{v}{c^2}x')(c - v)}(t - \frac{v}{c^2}x)^2 .$$

Let's solve this expression relative to t' taking into account that:

$$t' = nT', \quad x' = nl' , \quad \lambda'/_{T'} = c .$$

We obtain:

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\left(t - \frac{v}{c^2}x\right). \qquad (36)$$

Hence, the relation between measurement results of time interval in two reference frames will be the following:

$$\Delta t' = t_2' - t'_1 = \gamma(t_2 - t_1) = \gamma \Delta t . \qquad (37)$$

Here Δt' represents the time interval between two events happening in the same point x, in proper system; Δt is the same interval measured in system, relatively which the wave-object moves.

Expressions (30) and (37) represent the Lorentz transformations. From the proofs above mentioned, it is possible to make the following conclusions:

- the Lorentz transformations are not linked to presence or absence of any waves medium-carrier, in other words, presence of the waves medium-carrier does not contradict Lorentz transformations;
- the Lorentz transformations can be presented as algebra (group), defined on the set of functions of view (5) and (6).

Last statement was discussed by us in more detail in works [7, 8].

2.4. The transformation of parameters of traveling waves at frame changing.

For the analysis of interactions between waves, it is necessary to find else the transformation rules for frequency and wave number of traveling wave at transition from one undular frame in another. Let's consider a traveling wave described by expression:

$$a = A\cos(\omega t - kx), \tag{38}$$

in two states, with different values of phases:

$$\varphi_0 = \omega t_0 - kx_0 \tag{39}$$

and

$$\varphi_1 = \omega t_1 - kx_1 . \tag{40}$$

If

$$\varphi_1 - \varphi_0 = 2\pi n ,$$

where n - integer, the difference

$$\frac{\varphi_1}{2\pi} - \frac{\varphi_0}{2\pi} = n \tag{41}$$

describes the number of periods between states with phases φ_1 and φ_0. Taking into account (39) and (40), it is possible to copy expression (41):

$$(\omega t_1 - kx_1) - (\omega t_0 - kx_0) = 2\pi n ,$$

or:

$$\omega\left(t_1 - t_0 - \frac{x_1 - x_0}{c} \right) = 2\pi n . \tag{42}$$

As n is simply a number, it should not depend on reference system. Hence, for any other frame (marked by an accent) the same expression will be valid:

$$\omega'\left(t_1' - t_0' - \frac{x_1' - x_0'}{c} \right) = 2\pi n . \tag{43}$$

Equating the left-hand parts of expressions (42) and (43), we shall obtain:

$$\omega\left(t_1 - t_0 - \frac{x_1 - x_0}{c} \right) = \omega'\left(t_1' - t_0' - \frac{x_1' - x_0'}{c} \right). \tag{44}$$

Taking into account relations (30), (36), and also (31) we shall copy (44):

$$\omega\left(t_1 - t_0 - \frac{x_1 - x_0}{c} \right) = \omega'\gamma\left(t_1 - t_0 - \frac{v}{c^2}(x_1 - x_0) - \frac{x_1 - x_0 - v(t_1 - t_0)}{c} \right).$$

We consider the change of a phase from φ_1 up to φ_2 in a fixed point x_1=x_0. This corresponds to passage of a wave packet in interval of time t_1-t_0. Expression (44) will take the view:

$$\omega = \omega'\gamma(1 + \beta),$$

or:

$$\omega = \omega' \sqrt{\frac{1+\beta}{1-\beta}} \ .$$

This expression allows to obtain the expressions for frequencies of travelling components of moving wave-object:

$$\omega_1 = \omega \sqrt{\frac{1-\beta}{1+\beta}} \qquad \omega_2 = \omega \sqrt{\frac{1+\beta}{1-\beta}} \ , \tag{45}$$

where ω is the frequency of wave-object in the proper reference frame.

Taking into account, that $\omega = ck$, we shall obtain similar expressions for wave numbers:

$$k_2 = k \sqrt{\frac{1+\beta}{1-\beta}} \qquad k_1 = k \sqrt{\frac{1-\beta}{1+\beta}} \tag{46}$$

Thus, the formulas, which relate the values of frequencies and wave numbers of a traveling wave in different undular systems, coincide with the relativistic formulas for a longitudinal Doppler effect.

If to insert expressions (45) in (8), we shall obtain the relation between the oscillation frequency of wave-object in proper system ω and oscillation frequency of wave-object ω', measured in marked system:

$$\omega' = \frac{\omega}{2}\left(\sqrt{\frac{1-\beta}{1+\beta}} + \sqrt{\frac{1+\beta}{1-\beta}}\right) = \frac{\omega}{\sqrt{1-\beta^2}} \ . \tag{47}$$

3. Kinematics of interaction of stables waves-objects.

It is accepted to consider, that in linear media the principle of superposition is observed, and the waves practically do not interact among themselves. When talking about interactions between waves, it is meant usually that the medium is either nonlinear, or nonuniform. We shall demonstrate, that, if to describe process in undular frames, along with the solution, which corresponds to the superposition principle, it is possible to obtain the solution, in correspondence with which, the waves can interact among themselves in linear, homogeneous medium.

It is known, that at interaction of waves, the spatially-time resonance of waves takes place, the conditions of which have the view:

$$\sum \Delta\omega_i = 0 \ \sum \Delta k_i = 0 \ ;, \tag{48}$$

Where $\Delta\omega_i$ and Δk_i are the changes of frequencies and wave numbers of interacting waves. That is, at interaction of two waves, the changes of their frequencies will be identical in value, but will have opposite signs:

$$\Delta\omega_1 = -\Delta\omega_2 \ . \tag{49}$$

Expressions (48), (49) are the consequence that, time intervals of interaction for the both waves are identical. For wave numbers of two interacting waves also it is possible to note:

$$\Delta k_1 = -\Delta k_2 \ . \tag{50}$$

However in case of wave numbers it is necessary to mean, that they are vectors. The vectors of wave numbers of traveling waves-component are directed to the opposite parties and have equal modulo in the proper frame of standing wave-object.

$$k_1 = -k_2 = k \ .$$

Therefore resulting wave number of a wave (2):

$$k_r = k_1 + k_2 ,$$

that is wave-object in a quiescence, is equal to zero. The wave-object, which moves, is described by expression:

$$a' = A'\sin\left(\frac{k_1 + k_2}{2}r' - \frac{\omega_1 - \omega_2}{2}t'\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t' + \frac{k_1 - k_2}{2}r'\right) \quad . \tag{51}$$

Here wave numbers of traveling waves-component k_1 and k_2 do not have equal modulo. The resulting wave number of wave-object k_r is equal to half from difference of wave numbers of waves-components. From here, in view of expressions (46), for a resulting wave number of moving wave-object it is possible to note:

$$k_r = \frac{k}{2}\left(\sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1-\beta}{1+\beta}}\right) = \frac{\beta k}{\sqrt{1-\beta^2}} . \tag{52}$$

Thus, expression (50) for resulting wave numbers of two waves-objects, will gain the form:

$$\Delta k_{r1} = -\Delta k_{r2} \tag{53}$$

In the above-mentioned deductions we did not impose any restrictions on amplitude. In particular, the amplitude can be some spherically symmetric function from radius r. Then instead of expression (2) will be the wave-object, described by the formula

$$a = A(r)\sin kr \sin \omega t , \tag{54}$$

and representing the superposition of two waves, convergent and divergent, without a singular point in center.

Let's assume, that there exists a stable spherical wave of the view (54). It means this wave maintains the shape in the proper frame at interactions with another wave. We will solve a problem about interaction of two such waves, using undular frames.

In laboratory system, we shall designate: β_1 and β_2 - the normalized velocities of centers of waves-object along the axis x before interaction, and β_1', β_2' - the same velocities after interaction.

Let the frequencies of interacting waves be equal accordingly to ω_{01} and ω_{02} in proper frames. Then, according to formula (47), the frequencies of waves-objects measured in laboratory system up to interaction of waves, will be:

$$\omega_1 = \omega_{01}\frac{1}{\sqrt{1-\beta_1^2}} \quad \text{and} \quad \omega_2 = \omega_{02}\frac{1}{\sqrt{1-\beta_2^2}} , \tag{55}$$

and after interaction:

$$\omega_1' = \omega_{01}\frac{1}{\sqrt{1-(\beta_1')^2}} \text{ and } \omega_2' = \omega_{02}\frac{1}{\sqrt{1-(\beta_2')^2}} . \tag{56}$$

In this case expression (49) is possible to write as

$$\omega_1 - \omega_1' = \omega_2' - \omega_2 . \tag{57}$$

By inserting (55) and (56) in (57), we shall obtain the equation for frequencies:

$$\frac{\omega_{01}}{\sqrt{1-\beta_1^2}} - \frac{\omega_{01}}{\sqrt{1-(\beta_1')^2}} = \frac{\omega_{02}}{\sqrt{1-(\beta_2')^2}} - \frac{\omega_{02}}{\sqrt{1-\beta_2^2}} \tag{58}$$

Similarly, following expression (52), for resulting wave numbers of waves-objects, before interaction, it is possible to note:

$$k_{r1} = \frac{\beta_1 k_{01}}{\sqrt{1-\beta_1^2}} , \; k_{r2} = \frac{\beta_2 k_{02}}{\sqrt{1-\beta_2^2}} , \tag{59}$$

and after interaction:

$$k_{r1}'=\frac{\beta_1' k_{01}}{\sqrt{1-(\beta_1')^2}}\,,\quad k_{r2}'=\frac{\beta_2' k_{02}}{\sqrt{1-(\beta_2')^2}}\,. \qquad (60)$$

We shall copy equation (53) for wave numbers as

$$k_1-k_1'=k_2'-k_2$$

now we shall insert (59) and (60):

$$\frac{\beta_1 k_{01}}{\sqrt{1-\beta_1^2}}-\frac{\beta_1' k_{01}}{\sqrt{1-(\beta_1')^2}}=\frac{\beta_2' k_{02}}{\sqrt{1-(\beta_2')^2}}-\frac{\beta_2 k_{02}}{\sqrt{1-\beta_2^2}} \qquad (61)$$

Thus, we have obtained a system from two equations, (58) and (61), characterizing the process of interaction between two quasi-standing waves-objects. Taking into account that $k=\omega/c$, this system is possible to rewrite as:

$$\frac{\omega_{01}}{\sqrt{1-\beta_1^2}}+\frac{\omega_{02}}{\sqrt{1-\beta_2^2}}=\frac{\omega_{01}}{\sqrt{1-(\beta_1')^2}}+\frac{\omega_{02}}{\sqrt{1-(\beta_2')^2}} \qquad (62)$$

$$\frac{\omega_{01}\beta_1}{\sqrt{1-\beta_1^2}}+\frac{\omega_{02}\beta_2}{\sqrt{1-\beta_2^2}}=\frac{\omega_{01}\beta_1'}{\sqrt{1-(\beta_1')^2}}+\frac{\omega_{02}\beta_2'}{\sqrt{1-(\beta_2')^2}}\,. \qquad (63)$$

In this system from two equations, we shall consider as unknowns the normalized velocities of waves-objects after interaction.

The combined equations (62) and (63) have two pairs of the solutions. The first pair is trivial solution:

$$\beta_1'=\beta_1;\ \beta_2'=\beta_2\ . \qquad (64)$$

In correspondence with these solutions, the waves pass one through another, without any changes. It means expressions (64) describes the wave superposition. As usual these solutions are considered single possible for ideal medium.

The nontrivial solutions of combined equations (62) and (63) will be the expressions:

$$\beta_1'=\frac{A}{BD} \qquad (65)$$

$$\beta_2'=\frac{E}{FD} \qquad (66)$$

Here we used the following designation:

$$\begin{aligned}A=&\left[\beta_2\omega_{02}\left(1-\beta_1^2\right)\sqrt{1-\beta_2^2}\,\right]\left[3\beta_1\omega_{01}^2\left(1-\beta_2^2\right)-\omega_{02}^2\left(\beta_1+\beta_1\beta_2^2-2\beta_2\right)\right]+\\&+2\beta_2\omega_{01}\omega_{02}^2\sqrt{1-\beta_1^2}\left(\beta_1+\beta_2-\beta_1\beta_2^2-\beta_1^2\beta_2-\beta_2^3\right)+\\&+\beta_1^2\omega_{01}\sqrt{1-\beta_1^2}\left[\omega_{01}^2\left(1-2\beta_2^2+\beta_2^4\right)+\omega_{02}^2\left(3\beta_2^4-1\right)\right]\end{aligned} \qquad (67)$$

$$B=2\beta_1\beta_2\omega_{02}^2+\beta_2^2\left(\omega_{01}^2-\omega_{02}^2\right)-2\omega_{01}\omega_{02}\sqrt{1-\beta_1^2}\sqrt{1-\beta_2^2}-\omega_{01}^2-\omega_{02}^2 \qquad (68)$$

$$D=\beta_1\omega_{01}\sqrt{1-\beta_1^2}\left(\beta_2^2-1\right)+\beta_2\omega_{02}\sqrt{1-\beta_2^2}\left(\beta_1^2-1\right) \qquad (69)$$

$$\begin{aligned}E=&\left[\beta_1\omega_{01}\left(1-\beta_2^2\right)\sqrt{1-\beta_1^2}\,\right]\left[3\beta_2\omega_{01}^2\left(1-\beta_1^2\right)-\omega_{01}^2\left(\beta_2+\beta_1^2\beta_2-2\beta_1\right)\right]+\\&+2\beta_1\omega_{01}^2\omega_{02}\sqrt{1-\beta_2^2}\left(\beta_1+\beta_2-\beta_1^2\beta_2-\beta_1\beta_2^2-\beta_1^3\right)+\\&+\beta_2^2\omega_{02}\sqrt{1-\beta_2^2}\left[\omega_{02}^2\left(1-2\beta_1^2+\beta_1^4\right)+\omega_{01}^2\left(3\beta_1^4-1\right)\right]\end{aligned} \qquad (70)$$

$$F = 2\beta_1\beta_2\omega_{01}^2 - \beta_1^2\left(\omega_{01}^2 - \omega_{02}^2\right) - 2\omega_{01}\omega_{02}\sqrt{1-\beta_1^2}\sqrt{1-\beta_2^2} - \omega_{01}^2 - \omega_{02}^2 \tag{71}$$

We shall show that at

$$\beta = \frac{v}{c} << 1$$

from expressions (65) and (66) the formulas follow which coincide with the formulas describing the collision of two bodies in classical mechanics. For this purpose, in formulas (65) - (71) there is taken into account only terms having the least power of β, and β is neglected in comparison with unity. In this case expression (65) is conversed in the formula:

$$\beta_1' = \frac{\beta_2\omega_{02}\left(3\beta_1\omega_{01}^2 - \beta_1\omega_{02}^2 + 2\beta_2\omega_{02}^2\right)}{\left(2\omega_{01}\omega_{02} + \omega_{01}^2 + \omega_{02}^2\right)\left(\beta_1\omega_{01} + \beta_2\omega_{02}\right)} + \\ + \frac{2\beta_2\omega_{01}\omega_{02}^2\left(\beta_1 + \beta_2\right) + \beta_1^2\omega_{01}\left(\omega_{01}^2 - \omega_{02}^2\right)}{\left(2\omega_{01}\omega_{02} + \omega_{01}^2 + \omega_{02}^2\right)\left(\beta_1\omega_{01} + \beta_2\omega_{02}\right)}. \tag{72}$$

After a series of transformations expression (72) can be reduced to the form:

$$\beta_1' = \frac{\beta_1\left(\omega_{01} - \omega_{02}\right) + 2\beta_2\omega_{02}}{\left(\omega_{01} + \omega_{02}\right)}. \tag{73}$$

In a similar manner, from solution (66) we shall obtain the expression

$$\beta_2' = \frac{\beta_2\left(\omega_{02} - \omega_{01}\right) + 2\beta_1\omega_{01}}{\left(\omega_{01} + \omega_{02}\right)}. \tag{74}$$

By multiplying formulas (73) and (74) on c^2 and we shall obtain the expressions

$$v_1' = \frac{v_1\left(\omega_{01} - \omega_{02}\right) + 2v_2\omega_{02}}{\left(\omega_{01} + \omega_{02}\right)} \tag{75}$$

$$v_2' = \frac{v_2\left(\omega_{02} - \omega_{01}\right) + 2v_1\omega_{01}}{\left(\omega_{01} + \omega_{02}\right)}. \tag{76}$$

If we change the frequencies by masses in expressions (75) and (76), the formulas will be transformed in formulas, which describe the elastic collision of two mechanical bodies. Thus, description of process in undular frames, leads us to the conclusion, that the stable standing waves can interact as mechanical particles.

4. Interaction between standing and traveling wave

Let's suppose that the stable wave-object is situated in laboratory system and is described before interaction by expression (54). Its projection to the axis x is:

$$a = A(x)\sin kx \sin \omega t. \tag{77}$$

We remind, that the stable wave-object always remains invariant in a proper frame. That is, in proper frame it will be described by expression (77) or (54) always.

Incident traveling wave of the form

$$a_i = A\cos\left(\omega_i t - k_i x\right) \tag{78}$$

will try to deform wave-object (77), but this is not compatible with stability of the latter. Therefore wave-object (77) cannot remain immovable in laboratory system and will begin to move. Hence, in laboratory system it will be described already by expression for moving wave-object:

$$a' = A'(x)\sin\left(\frac{k_1 + k_2}{2}x' - \frac{\omega_1 - \omega_2}{2}t'\right)\sin\left(\frac{\omega_1 + \omega_2}{2}t' + \frac{k_1 - k_2}{2}x'\right). \tag{79}$$

Thus, after interaction with a traveling wave, the wave-object will be described by expression (77) in proper system, and expression (79) in laboratory system.

We will found the wave-object velocity after interaction. We mark the frequency of wave-object before interaction ω, after interaction - ω', and frequencies of incident wave - ω_i and ω_i' respectively. Let's insert it in (49):

$$\omega_i - \omega_i' = \omega' - \omega .$$

At total absorption of traveling wave, its frequency after interaction $\omega_i' = 0$. Hence

$$\omega_i = \omega' - \omega .$$

In view of expression (47):

$$\omega_i = \omega\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right).$$

By solving this equation, we shall obtain velocity, gained by wave-object as a result of interaction with the incident wave.

$$\beta = \pm\frac{\sqrt{2\omega\omega_i + \omega_i^2}}{\omega + \omega_i}.$$

We note, that the similar formula describes the interaction of a light quantum with an elementary particle.

Let's consider now the case, when the front of incident wave is non-uniform. It means, in expression (78) $A = A_i(y)$. In this case the incident wave acts asymmetrically in relation to center of wave-object, and after interaction the wave-object will move under some angle ψ relatively to axis x. As the wave numbers of interacting waves vary by the same value, the wave vector of the incident wave also will deviate by some angle φ (fig. 2).

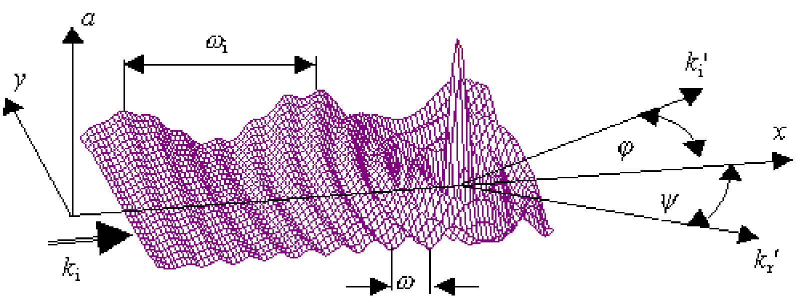

Figure 2. The wave-object (54), under action of a non-uniform travelling wave in two-dimensional representation

Let us suppose, the wave-object is situated in laboratory system before interaction, and is described by expression (54) (figure 1), its frequency is ω, and resulting wave number - k_r. The frequency of incident wave (78) before interaction is ω_i, and its wave number is k_i. The same values after interaction we shall mark by an accent.

As a result of interaction the frequency of wave-object will vary by value $\Delta\omega = \omega' - \omega$, or, taking into account (47):

$$\Delta\omega = \omega\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right). \qquad (80)$$

As was marked above, in the proper frame of wave-object, the wave numbers vectors of traveling waves-components are directed to opposite parties and have equal modulo. Therefore, in a quiescence, the resulting wave number of wave-object

$$k_r = 0.$$

After interaction the wave-object will gain some velocity v, and will be described by expression (79), in which the wave numbers of components now do not have equal modulo. Therefore the resulting wave number of wave-object k_r' will be determined in correspondence with expression (52):

$$k_r' = \frac{\beta k_r}{\sqrt{1-\beta^2}}.$$

Thus, the wave number of wave-object will change in result of interaction on:

$$\Delta k_r = k_r' - k_r = \frac{\beta k_r}{\sqrt{1-\beta^2}}$$

As $k = \omega/c$, with Δk_r it is possible to link some frequency $\Delta\omega_r$,

$$\Delta\omega_r = \frac{\omega_1 - \omega_2}{2} = \frac{\beta\omega}{\sqrt{1-\beta^2}},$$

taking into account (47):

$$\Delta\omega_r = \omega'\beta .$$

Then the difference:

$$(\omega')^2 - \omega^2 = (\omega')^2 - (\omega')^2(1-\beta^2) = (\omega')^2\beta^2,$$

or

$$(\omega')^2 - \omega^2 = (\Delta\omega_r)^2 . \qquad (81)$$

As was marked above, the frequencies of interacting waves vary by the same value

$$\Delta\omega = -\Delta\omega_i$$

or

$$\omega' - \omega = \omega_i - \omega_i' . \qquad (82)$$

As the wave number is a vector, the principle of identical change of wave numbers for interacting waves should be noted separately for projections on each axis:

$$\Delta k_r \cos\varphi = k_i - k_i'\cos\varphi , \qquad (83)$$

$$\Delta k_r \sin\varphi = k_i'\sin\varphi . \qquad (84)$$

Let's raise to the second power expression (83) and (84) and sum it:

$$(\Delta k_r)^2 = k_i^2 + (k_i')^2 - 2k_i k_i'\cos\varphi .$$

By dividing this expression by c^2, we shall obtain

$$(\Delta\omega_r)^2 = \omega_i^2 + (\omega_i')^2 - 2\omega_i\omega_i'\cos\varphi . \qquad (85)$$

We shall copy expression (82) as:

$$(\omega')^2 = (\omega_i - \omega_i' + \omega)^2 = \omega_i^2 + (\omega_i')^2 - 2\omega_i\omega_i' + 2\omega(\omega_i - \omega_i') + \omega^2, \qquad (86)$$

now we shall subtract (85) from (86):

$$(\omega')^2 - (\Delta\omega_r)^2 - \omega^2 = -2\omega_i\omega_i'(1-\cos\varphi) + 2\omega(\omega_i - \omega_i'). \qquad (87)$$

In correspondence with expression (81), the left-hand part of (87) is equal to zero, so:

$$\omega_i\omega_i'(1-\cos\varphi) = \omega(\omega_i - \omega_i'),$$

or:

$$\frac{1-\cos\varphi}{\omega} = \frac{\omega_i - \omega_i'}{\omega_i\omega_i'} .$$

That is equivalent to expression:

$$\frac{1-\cos\varphi}{\omega} = \frac{1}{\omega_i'} - \frac{1}{\omega_i}, \qquad (88)$$

Taking into account, that $\omega = \frac{2\pi c}{\lambda}$, expression (88) will be copied as:

$$\lambda_i - \lambda_i' = \lambda(1 - \cos\varphi)$$

Where: λ_i and λ_i' are wave length of incident and dispersed traveling waves, and λ is wave length of wave-object "reposing" in laboratory system. If we designate the change of length for the traveling wave, resulted from its interaction with the standing wave as $\Delta\lambda_i = \lambda_i' - \lambda_i$, we shall obtain:

$$\Delta\lambda_i = 2\lambda \sin^2\frac{\varphi}{2}. \qquad (89)$$

This is the Compton's formula, which describes the interaction between electron and light quantum. We have obtained it without use of the concepts of mass, impulse or energy. Thus, the Compton effect can be presented as kinematic effect of interaction between the standing stable wave and traveling wave.

5. Conclusions

1. In the deductions presented above, any medium-carrier properties of waves do not appear, thus these deductions are valid for all wave types irrespective of medium. It signifies, that the Lorentz transformations are not interlinked on the presence or absence of the wave medium-carrier and can be considered as rules of some algebra (or group), which is defined on set of wave functions.
2. The application of undular frames allows us to obtain not only a trivial solutions relevant to a principle of superposition, but also solutions, which describe the interactions between waves as between mechanical particles.
3. At wave interaction their frequency varies. At interaction between the stable standing wave and the traveling wave, the quantization of traveling wave takes place.

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