TWO-DIMENSIONAL CAVITY POLARITONS UNDER THE INFLUENCE OF STRONG PERPENDICULAR MAGNETIC AND ELECTRIC FIELDS

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(Received January 22, 2015)

Abstract

The properties of two-dimensional (2D) cavity polaritons subjected to the action of strong perpendicular magnetic and electric fields giving rise to the Landau quantization (LQ) of the 2D electrons and holes accompanied by the Rashba spin-orbit coupling (RSOC) and by the Zeeman splitting (ZS) have been investigated. A strong magnetic field, where the electron and the hole cyclotron energy frequencies are greater than the binding energy of the 2D Wannier-Mott excitons, completely reconstructs it transforming into a magnetoexciton, the structure of which is determined by the Lorentz force rather than by the Coulomb electron–hole (e–h) interaction.

We predict drastic changes in the optical properties of the cavity polaritons including those in the state of Bose-Einstein condensation. The main of them is the existence of a multitude of the polariton energy levels closely adjacent on the energy scale, their origin being related with the LQ of the electrons and holes. Most of these levels exhibit nonmonotonous dependences on magnetic field strength *B* with overlapping and intersections. More so, the selection rules for the band-to-band optical quantum transitions, as well as the quantum transitions from the ground state of the crystal to the magnetoexciton states, essentially depend on numbers n_e and n_h of the LQ levels of the e–h pair forming the magnetoexciton. By slowly changing the external magnetic and electric fields, it is possible to change the lowest polariton energy level, its oscillator strength, the probability of the quantum transition, and the Rabi frequency of the polariton dispersion law. They depend on the relation between numbers n_e and n_h and can lead to dipole-active, quadrupole-active, or forbidden optical transitions. Our results are based on the exact solutions for the eigenfunctions and the eigenvalues of the Pauli-type Hamilonian with third order chirality terms and a nonparabolic dispersion law for heavy-holes and with first order chirality terms for electrons. They were obtained using the method proposed by Rashba [1].

We expect that these results will also determine the collective behavior of the cavity polaritons, for example, in the GaAs-type quantum wells embedded into a microcavity, which have recently revealed the phenomenon of the Bose-Einstein condensation in the state of the thermodynamic quasi-equilibrium but in the absence of a strong perpendicular magnetic field.

1. Introduction

The aim of this study is to determine the properties of the two-dimensional (2D) polaritons arising in the frame of a quantum well (QW) embedded into a microcavity and subjected to the action of a strong perpendicular magnetic field, giving rise to the Landau quantization (LQ) of the 2D electrons and holes accompanied by the Rashba spin–orbit coupling (RSOC) and Zeeman splitting (ZS) effect. In the case of free electron–hole (e–h) pairs and a band-to-band quantum transition, the magnetic and electric fields with arbitrary intensities were considered; these aspects are discussed in the second section of this paper. The properties of 2D magnetoexcitons are investigated under the condition of a strong magnetic field when cyclotron energies frequencies ω_{ci} with $i = e, h$ are greater than the binding energy of the 2D Wannier–Mott excitons and magnetic length l_0 is smaller than the exciton Bohr radius. The binding energy of the 2D magnetoexcitons is obtainer taking into account the ionization potential determined by the Coulomb interaction in the frame of the e–h pair.

In section two of our paper, the wave functions and the energy levels of the 2D electrons and heavy-holes are discussed. The calculations of the electron–electron Coulomb interaction are conducted in the third section using the spinor-type conduction and valence electron wave functions describing the LQ accompanied by the RSOC and by the ZS effects. The magnetoexciton energy levels arising from different combinations of the electron and hole states are investigated.

The fourth section is focused on the electron–radiation interaction in the frame of the e–h system confined on the 2D layer and the electromagnetic field arbitrarily propagating in the three-dimensional (3D) space as regards the 2D layer. The corresponding Hamiltonian describing the magnetoexciton-photon interaction is deduced. The formation of magnetoexciton-polaritons in a microcavity is discussed. It is the main goal of our paper. The conclusions are made in the fifth section.

2. 2D electrons and holes under the influence of the perpendicular magnetic and electric fields.

The Hamiltonians describing the LQ, RSOC, and ZS effect involving 2D electrons and holes were deduced in [1–6]. They have the form

$$H_e = \hbar\omega_{ce}\left\{\left(a^+a+\frac{1}{2}\right)\hat{I} + i\sqrt{2}\alpha\begin{vmatrix}0 & a\\ -a^+ & 0\end{vmatrix} + Z_e\hat{\sigma}_z\right\}$$

$$H_h = \hbar\omega_{ch}\left\{\left[\left(a^+a+\frac{1}{2}\right)+\delta\left(a^+a+\frac{1}{2}\right)^2\right]\hat{I} + i2\sqrt{2}\beta\begin{vmatrix}0 & \left(a^+\right)^3\\ -\left(a\right)^3 & 0\end{vmatrix} - Z_n\hat{\sigma}_z\right\} \tag{1}$$

Here, Bose-type operators a^+, a generating Fock states $\left|m\right\rangle$ were introduced and the following notation was used:

$$Z_i = \frac{g_i \mu_B B}{2\hbar \omega_{ci}} = \frac{g_i m_i}{4 m_0}, \quad \omega_{ci} = \frac{eB}{m_i c}, \quad i = e, h.$$
$$\hat{I} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \quad \hat{\sigma}_z = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}, \quad \mu_B = \frac{e\hbar}{2 m_0 c}, \quad e = |e| > 0.$$
$$\alpha = \frac{8 \cdot 10^{-3} x}{\sqrt{y}}, \quad \beta = 1.062 \cdot 10^{-2} x \sqrt{y},$$
$$\delta = 10^{-4} C x y, \quad B = y\mathrm{T}, \quad E_z = x \frac{\mathrm{kV}}{\mathrm{cm}} \tag{2}$$

where ω_{ci} are the cyclotron frequencies, Z_i are the Zeeman parameters proportional to g -factors g_i and to effective masses m_i of the electrons and holes, whereas m_0 is the bare electron mass; δ is the nonparabolicity (NP) of the heavy-hole dispersion law, α and β are the parameters of the chirality terms, which are of the first order in the case of electrons [1] and of the third order in the case of heavy holes [7–8].

The solutions of these equations were chosen in the dimensionless forms. For the electron case, we have

$$\mathscr{H}_e = \frac{H_e}{\hbar \omega_{ce}}; \quad \mathscr{H}_e |\psi_e\rangle = \varepsilon |\psi_e\rangle; \quad |\psi_e\rangle = \begin{vmatrix} |f_1\rangle \\ |f_2\rangle \end{vmatrix}$$
$$|f_1\rangle = \sum_n a_n |n\rangle; \quad |f_2\rangle = \sum_n b_n |n\rangle; \quad \sum_n |a_n|^2 + \sum_n |b_n|^2 = 1 \tag{3}$$

In the coordinate representation, the wave functions are as follows:

$$|\psi_m^{\pm}(\vec{r})\rangle = \frac{e^{ipx}}{\sqrt{L_x}} \begin{vmatrix} a_m^{\pm} \varphi_m(\eta) \\ b_{m+1}^{\pm} \varphi_{m+1}(\eta) \end{vmatrix}; \quad \eta = \frac{y}{l} - pl \tag{4}$$

where L_x is the length of the layer. These states were obtained firstly by Rashba [1] and are repeated here including Zeeman coefficient Z_e.

Along with the solutions $|\psi_m^{\pm}\rangle$ with $m \geq 0$, there exists another solution with $b_0 = 1$ of the type

$$\varepsilon_0 = \frac{1}{2} - Z_e; \quad |\psi_0\rangle = \begin{vmatrix} 0 \\ |0\rangle \end{vmatrix}; \quad |\psi_0(\vec{r})\rangle = \frac{e^{ipx}}{\sqrt{L_x}} \begin{vmatrix} 0 \\ \varphi_0(\eta) \end{vmatrix} \tag{5}$$

which is orthogonal to any solutions (4).

In energy units, the energy spectrum is as follows:

$$E_m^{\pm} = \hbar \omega_{ce} \varepsilon_m^{\pm}, \quad m \geq 0,$$
$$E_0 = \hbar \omega_{ce} \varepsilon_0 \tag{6}$$

Below, we will consider only two lowest Landau levels (LLs) for conduction electrons, namely state $|\psi_0^-\rangle$ (4) and $|\psi_0\rangle$ (5).

The heavy-hole Hamiltonian in a dimensionless form has the form

$$\mathscr{H} = \frac{H_h}{\hbar\omega_{ch}} = \left\{ \left[\left(a^+a + \frac{1}{2} \right) + \delta \left(a^+a + \frac{1}{2} \right)^2 \right] \hat{I} + i2\sqrt{2}\beta + \begin{vmatrix} 0 & \left(a^+\right)^3 \\ -(a)^3 & 0 \end{vmatrix} - Z_h \hat{\sigma}_z \right\};$$

$$\mathscr{H} \left| \psi \right\rangle = \varepsilon \left| \psi \right\rangle$$

$$\left| \psi \right\rangle = \begin{vmatrix} \left| f_1 \right\rangle \\ \left| f_2 \right\rangle \end{vmatrix}; \quad \left| f_1 \right\rangle = \sum_{n \geq 0} c_n \left| n \right\rangle; \quad \left| f_2 \right\rangle = \sum_{n \geq 0} d_n \left| n \right\rangle \tag{7}$$

The respective wave functions have the form

$$\left| \psi_m^{\pm} \right\rangle = \begin{vmatrix} c_m^{\pm} \left| m \right\rangle \\ d_{m-3}^{\pm} \left| m-3 \right\rangle \end{vmatrix}, \quad \left| \psi_m^{\pm}(\vec{r}) \right\rangle = \frac{e^{iqx}}{\sqrt{L_x}} \begin{vmatrix} c_m^{\pm} \varphi_m(\eta) \\ d_{m-3}^{\pm} \varphi_{m-3}(\eta) \end{vmatrix}, \tag{8}$$

$$\eta = \frac{y}{l} + ql, \quad m \geq 3.$$

They obey the normalization and orthogonality conditions

$$\left\langle \psi_m^{\pm} \middle| \psi_m^{\pm} \right\rangle = \left| c_m^{\pm} \right|^2 + \left| d_{m-3}^{\pm} \right| = 1, \quad \left\langle \psi_m^{+} \middle| \psi_m^{-} \right\rangle = c_m^{+*} c_m^{-} + d_{m-3}^{+*} d_{m-3}^{-} = 0 \tag{9}$$

Along with solutions (8), there exist three other solutions with $m = 0,1,2$. They are:

$$c_0 = 1, \quad \left| \psi_0 \right\rangle = \begin{vmatrix} \left| 0 \right\rangle \\ 0 \end{vmatrix}, \quad \left| \psi_0(\vec{r}) \right\rangle = \frac{e^{iqx}}{\sqrt{L_x}} \begin{vmatrix} \varphi_0(\eta) \\ 0 \end{vmatrix},$$

$$\varepsilon_0 = \frac{1}{2} + \frac{\delta}{4} - Z_h,$$

$$c_1 = 1, \quad \left| \psi_1 \right\rangle = \begin{vmatrix} \left| 1 \right\rangle \\ 0 \end{vmatrix}, \quad \left| \psi_1(\vec{r}) \right\rangle = \frac{e^{iqx}}{\sqrt{L_x}} \begin{vmatrix} \varphi_1(\eta) \\ 0 \end{vmatrix}, \tag{10}$$

$$\varepsilon_1 = \frac{3}{2} + \frac{9}{4}\delta - Z_h,$$

$$c_2 = 1, \quad \left| \psi_2 \right\rangle = \begin{vmatrix} \left| 2 \right\rangle \\ 0 \end{vmatrix}, \quad \psi_2(\vec{r}) = \frac{e^{iqx}}{\sqrt{L_x}} \begin{vmatrix} \varphi_2(\eta) \\ 0 \end{vmatrix},$$

$$\varepsilon_2 = \frac{5}{2} + \frac{25}{4}\delta - Z_h$$

All of them are orthogonal to previous solutions (8).

3. The Coulomb electron–electron interaction and magnetoexcitons

The three LLLs for 2D heavy-holes (h, R_j) with $j = 1,2,3$ were combined with two LLLs for 2D conduction electrons (e, R_i) with $i = 1,2$ giving rise to six 2D magnetoexciton states F_n with $n = 1,2 \ldots 6$ [3, 4]. To calculate their ionization potentials, the Hamiltonian of the Coulomb electron–electron interaction under the conditions of LQ, RRSOC, and ZS is required. It was deduced in [3, 4] taking into account only the first two conditions, namely LQ and RSOC. Below, we will generalize those results adding the third condition, namely the ZS effects. The more so, it can be done because the Pauli Hamiltonians containing the Zeeman effects are represented by operators $\pm g_i \frac{\mu_B B}{2}\hat{\sigma}_z$, where $\hat{\sigma}_z$ have only diagonal matrix elements. For the electron–hole (e–h) pair in the concrete combination (R_i, ε_m^-) the Hamiltonian of the Coulomb interaction has the form

$$H_{coul}(R_i,\varepsilon_m^-) = \frac{1}{2}\sum_{\vec{a}} \Big\{ W_{e-e}(R_i;\vec{Q})\Big[\hat{\rho}_e(R_i;\vec{Q})\hat{\rho}_e(R_i,-\vec{Q}) - \hat{N}_e(R_i)\Big]$$
$$+W_{h-h}(\varepsilon_m^-;\vec{Q})\Big[\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q}) - \hat{N}_h(M_h,\varepsilon_m^-)\Big]$$
$$-2W_{e-h}(R_i,\varepsilon_m^-;\vec{Q})\hat{\rho}_e(R_i;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;-\vec{Q})\Big\} \tag{11}$$

Here, the electron and hole density operators are:

$$\hat{\rho}_e(R_i;\vec{Q}) = \sum_t e^{iQ_y t l_0^2} a^+_{R_i,t+\frac{Q_x}{2}} a_{R_i,t-\frac{Q_x}{2}}$$
$$\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q}) = \sum_t e^{-iQ_y t l_0^2} b^+_{M_h,\varepsilon_m^-,t+\frac{Q_x}{2}} b_{M_h,\varepsilon_m^-,t-\frac{Q_x}{2}} \tag{12}$$
$$\hat{N}_e(R_i) = \hat{\rho}_e(R_i,0);\ \hat{N}_h(M_h,\varepsilon_m^-) = \hat{\rho}_h(M_h,\varepsilon_m^-;0)$$

Coefficients $W_{i-j}(Q)$ in the case of the electron state R_1 described by formula (4) with coefficients a_0^- and b_1^- and the hole states (M_h,ε_m^-) were deduced in [4]:

$$W_{e-e}(R_1;\vec{Q}) = W(Q)\Big(|a_0^-|^2 A_{0,0}(\vec{Q}) + |b_1^-|^2 A_{1,1}(\vec{Q})\Big)^2$$
$$W_{h-h}(\varepsilon_m^-;\vec{Q}) = W(\vec{Q})\Big(|d_{m-3}^-|^2 A_{m-3,m-3}(Q) + |c_m^-|^2 A_{m,m}(Q)\Big)^2,\ \mathrm{m}\ \geq 3$$
$$W_{e-h}(R1,\varepsilon_m^-;\vec{Q}) = W(\vec{Q})\Big(|a_0^-|^2 A_{0,0}(\vec{Q}) + |b_1^-|^2 A_{1,1}(\vec{Q})\Big)\times \tag{13}$$
$$\times\Big(|d_{m-3}^-|^2 A_{m-3,m-3}(\vec{Q}) + |c_m^-|^2 A_{m,m}(\vec{Q})\Big),\ m \geq 3$$

with the normalization conditions

$$|a_0^-|^2 + |b_1^-|^2 = 1 \quad |d_{m-3}^-|^2 + |c_m^-|^2 = 1 \quad m \geq 3 \tag{14}$$

The first five functions $A_{m,m}(\vec{Q})$ with $m \leq 4$ are:

$$A_{0,0}(\vec{Q}) = 1;\ A_{1,1}(\vec{Q}) = 1 - \frac{Q^2 l_0^2}{2},$$
$$A_{2,2}(\vec{Q}) = 1 - Q^2 l_0^2 + \frac{Q^4 l_0^4}{8},$$
$$A_{3,3}(\vec{Q}) = 1 - \frac{3}{2}Q^2 l_0^2 + \frac{3}{8}Q^4 l_0^4 - \frac{1}{48}Q^6 l_0^6, \tag{15}$$
$$A_{4,4}(\vec{Q}) = 1 - 2Q^2 l_0^2 + \frac{3}{4}Q^4 l_0^4 - \frac{Q^6 l_0^6}{12} + \frac{Q^8 l_0^8}{384}$$

Along with the electron state R_1, we will consider state R_2 described by formulas (5).

The combination of electron state R_2 with hole states ε_m^- gives rise to the e–h states $(eR_2, h, \varepsilon_m^-)$. Coefficients $W_{i-j}(R_2, \varepsilon_m^-; \vec{Q})$ can be obtained from coefficients $W_{i-j}(R_1, \varepsilon_m^-; \vec{Q})$ putting $b_1^- = 0$ in them as follows:

$$W_{i-j}\left(R_2, \varepsilon_m^-; \vec{Q}\right) = W_{i-j}\left(R_1, \varepsilon_m^-; \vec{Q}\right)\Big|_{b_1^- = 0}, \qquad i, j = e, h \tag{16}$$

The terms proportional to $\hat{N}_e(R_1)$ and $\hat{N}_h(M_h, \varepsilon_m^-)$ in (11) have coefficients $I_e(R_1)$ and $I_h(\varepsilon_m^-)$ describing the Coulomb self-actions of the electrons and holes. They are as follows:

$$I_e(R_1) = \frac{1}{2}\sum_{Q} W_{e-e}\left(R_1, \vec{Q}\right); \; I_h\left(\varepsilon_m^-\right) = \frac{1}{2}\sum_{Q} W_{h-h}\left(\varepsilon_m^-; \vec{Q}\right)$$
$$I_s\left(R_1, \varepsilon_m^-\right) = I_e(R_1) + I_h\left(\varepsilon_m^-\right) \tag{17}$$

To determine the binding energy of the magnetoexciton, its wave functions $\left|\psi_{ex}\left(F_i, \vec{K}\right)\right\rangle$ were obtained acting on vacuum state $|0\rangle$ by magnetoexciton creation operator $\psi_{ex}^{\dagger}(k_{||}, M_h, R_i, \varepsilon)$ constructed from electron and hole creation operators $a_{R_i,t}^{\dagger}$ and $b_{M_h,\varepsilon,t}^{\dagger}$, respectively, as follows:

$$\psi_{ex}^{\dagger}\left(\vec{k}_{||}, R_i, M_h, \varepsilon\right) = \frac{1}{\sqrt{N}}\sum_{t} e^{ik_y t l_0^2} a^{\dagger}_{R_i, t+\frac{k_x}{2}} b^{\dagger}_{M_h, \varepsilon; -t+\frac{k_x}{2}},$$
$$\left|\psi_{ex}\left(R_i; M_h, \varepsilon; \vec{k}_{||}\right)\right\rangle = \psi_{ex}^{\dagger}\left(\vec{k}_{||}, R_i, M_h, \varepsilon\right)|0\rangle \tag{18}$$

The vacuum state is determined by the equalities

$$a_{\xi,t}|0\rangle = b_{\xi,t}|0\rangle = 0. \tag{19}$$

Below, a concrete composition $F_i = \left(R_1, M_h, \varepsilon_m^-\right)$ with $m \geq 3$ of the electron and hole states will be considered.

The binding energy of the magnetoexciton is determined by the diagonal matrix element of Hamiltonian (11) calculated with wave function $\left|\psi_{ex}\left(F_i, \vec{k}\right)\right\rangle$:

$$\left\langle\psi_{ex}\left(F_i, \vec{k}\right)\right| H_{coul} \left|\psi_{ex}\left(F_i, \vec{k}\right)\right\rangle = -I_{ex}(F_i) + E\left(F_i, \vec{k}\right)$$
$$I_{ex}(F_i) = I_{ex}\left(R_1, \varepsilon_m^-\right) = \sum_{Q} W_{e-h}\left(R_1; \varepsilon_m^-; \vec{Q}\right)$$
$$E\left(F_i, \vec{k}\right) = E\left(R_1; \varepsilon_m^-; \vec{k}\right) = 2\sum_{Q} W_{e-h}\left(R_1; \varepsilon_m^-; \vec{Q}\right) sin^2\left(\frac{\left[\vec{k}\times\vec{Q}\right]_z l_0^2}{2}\right) \tag{20}$$
$$\lim_{k\to\infty} E\left(R_1; \varepsilon_m^-; \vec{k}\right) = I_{ex}\left(R_1; \varepsilon_m^-\right)$$

The binding energy of the magnetoexciton and its ionization potential, which has the opposite sign as compared with the binding energy, tend to zero if the wave vector k tends to infinity and the magnetoexciton is transformed into a free e–h pair.

4. Interaction of magnetoexcitons with the electromagnetic radiation and the formation of cavity polaritons

In [9–12], the Hamiltonians describing, using different approximations, the electron-radiation interaction in the system of two-dimensional coplanar electrons and holes accumulated in the semiconductor QW and subjected to the action of a strong perpendicular magnetic field giving rise to the LQ of their energy levels were deduced. Only the case of the interband optical quantum transitions with the creation or annihilation of one electron–hole pair in [9-12] was considered. The intraband quantum transitions were discussed in [13]. In [9], the exciton-cyclotron resonance and the optical orientation phenomena [14] arising under the influence of the circularly polarized laser radiation were studied without taking into account the RSOC. The Hamiltonian deduced in [9] in the e–h representation was transcribed in [10] so as to describe the magnetoexciton-photon interaction for the light arbitrarily propagating in the three-dimensional (3D) space as well as being confined in the microcavity. The dispersion law of the magnetoexciton-polariton in a microcavity was deduced. The dependence of the Rabi frequency on the magnetic field strength and the selection rule concerning the numbers of the LQ levels of the e–h pair engaged in the dipole-active and quadrupole-active transitions were determined [10]. The influence on the optical properties of the magnetoexcitons on the band-to-band quantum transitions of the RSOC arising due to the action of a supplementary electric field perpendicular to the plane of the QW was investigated in [3, 4, 11, 12]. The third order chirality terms in Hamiltonian (1) of the heavy hole induced by the electric field obliged one to introduce an additional NP term of the same origin in the heavy-hole dispersion law. The NP term, together with the chirality terms, essentially changes the dependence of the energy levels on the magnetic field strength. The role of the NP term is to prevent the unlimited deep penetration of the energy levels of the 2D heavy-hole, as well as of the e–h pair and the magnetoexciton, inside the energy band gap and contribute to the stability of the semiconductor band structure. In [3, 4, 11, 12], the effects related with the ZS were not discussed. This shortage is pieced out below. We will discuss the properties of magnetoexcitons and magnetoexciton polaritons taking into account the influence of a full set of four factors, such as LQ, RSOC, ZS, and NP. The Hamiltonian describing the magnetoexciton–photon interaction including the ZS effects has exactly the same form as in [11] with only one difference that coefficients $\bar{a}_0$, $\bar{b}_1$, $\bar{c}_m$, and $\bar{d}_{m-3}$ with $m \geq 3$ must be determined in [13] by the expressions containing nonzero Zeeman coefficients z_e and z_h. In previous papers [3, 4, 11, 12], these coefficients were absent.

Following formula (41) of [11], the Hamiltonian of the magnetoexciton–photon interaction has the form

$$
\begin{aligned}
\hat{H}_{e-rad} = & \left(-\frac{e}{m_0 l_0}\right) \sum_{\vec{k}(k_{\|},k_z)} \sum_{M_h=\pm 1} \sum_{i=1,2} \sum_{\varepsilon=\varepsilon_m,\bar{\varepsilon}_m} \sqrt{\frac{\hbar}{L_z \omega_{\vec{k}}}} \times \\
& \times \Big\{ P_{cv}(0) T\left(R_i,\varepsilon,\vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_{M_h}^{*}\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{\mp}\cdot\vec{\sigma}_{M_h}^{*}\right)\right]\hat{\Psi}_{ex}^{\dagger}(\vec{k}_{\|},M_h,R_i,\varepsilon) + \\
& + P_{cv}^{*}(0) T^{*}\left(R_i,\varepsilon,\vec{k}_{\|}\right)\left[\left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{\mp}\cdot\vec{\sigma}_{M_h}\right) + \left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_{M_h}\right)\right]\hat{\Psi}_{ex}(\vec{k}_{\|},M_h,R_i,\varepsilon) + \\
& + P_{cv}(0) T\left(R_i,\varepsilon,-\vec{k}_{\|}\right)\left[\left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{\mp}\cdot\vec{\sigma}_{M_h}^{*}\right) + \left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_{M_h}^{*}\right)\right]\hat{\Psi}_{ex}^{\dagger}(-\vec{k}_{\|},M_h,R_i,\varepsilon) + \\
& + P_{cv}^{*}(0) T^{*}\left(R_i,\varepsilon,-\vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_{M_h}\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{\mp}\cdot\vec{\sigma}_{M_h}\right)\right]\hat{\Psi}_{ex}(-\vec{k}_{\|},M_h,R_i,\varepsilon) \Big\}
\end{aligned}
\tag{21}
$$

It differs from Hamiltonian (9) of [10] by the more complicate coefficients $T\left(R_i, \varepsilon, \vec{k}_{\parallel}\right)$, which in turn now contain generalized coefficients a_0^-, b_1^-, c_m^-, and d_{m-3}^- given by the expressions determined in [13].

Hamiltonian (21) contains the creation and annihilation operators of magnetoexcitons $\hat{\Psi}_{ex}^{\dagger}(k_{\parallel}, M_h, R_i, \varepsilon)$, $\hat{\Psi}_{ex}(k_{\parallel}, M_h, R_i, \varepsilon)$ and photons $C_{\vec{k},\xi}^{\dagger}$, $C_{\vec{k},\xi}$. The former were determined by formula (18) and are characterized by in-plane wave vectors $k_{\parallel}$ by orbital projection M_h of the hole state in the frame of the p-type valence band and by quantum states R_i and ε of the electron and hole under the conditions of the LQ accompanied by the RSOC, ZS, and NP. Instead of quantum number M_h, circular polarization vector $\vec{\sigma}_{M_h}$ will be introduced. The photon operators depend on wave vectors $k = \vec{a}_3 k_z + k_{\parallel}$ arbitrary oriented in the 3D space, where $\vec{a}_3$ is an unit vector perpendicular to the layer, and on polarization label ξ, which takes two values—1 and 2—in the case of the light with linear polarizations $\vec{e}_{\vec{k},i}$ or the signs $\pm$ in the case of circular polarizations $\vec{\sigma}_{\vec{k}}^{\pm}$. The required denotations are:

$$
\begin{aligned}
& C_{\vec{k},\pm} = \frac{1}{\sqrt{2}}\left(C_{\vec{k},1} \pm iC_{\vec{k},2}\right), \ \left(C_{\vec{k},\pm}\right)^{\dagger} = \frac{1}{\sqrt{2}}\left(C_{\vec{k},1}^{\dagger} \mp iC_{\vec{k},2}^{\dagger}\right), \\
& \vec{\sigma}_{\vec{k}}^{\pm} = \frac{1}{\sqrt{2}}\left(\vec{e}_{\vec{k},1} \pm i\vec{e}_{\vec{k},2}\right), \ \left(\vec{e}_{\vec{k},1} \cdot \vec{k}\right) = 0, \ i = 1, 2, \\
& \sum_{i=1}^{2} C_{\vec{k},i}\vec{e}_{\vec{k},i} = C_{\vec{k},-}\vec{\sigma}_{\vec{k}}^{\pm} + C_{\vec{k},+}\vec{\sigma}_{\vec{k}}^{\mp}, \\
& \sum_{i=1}^{2} \left(C_{\vec{k},\pm}\right)^{\dagger}\vec{e}_{\vec{k},i} = \left(C_{\vec{k},-}\right)^{\dagger}\vec{\sigma}_{\vec{k}}^{\mp} + \left(C_{\vec{k},+}\right)^{\dagger}\vec{\sigma}_{\vec{k}}^{\pm}, \\
& \vec{\sigma}_{M_h} = \frac{1}{\sqrt{2}}(\vec{a}_1 \pm i\vec{a}_2), \ \vec{k}_{\parallel} = k_x\vec{a}_1 + k_y\vec{a}_2, \\
& \left(\vec{\sigma}_{M_h} \cdot \vec{a}_3\right) = 0, \ M_h = \pm 1,
\end{aligned}
\tag{22}
$$

Here $\vec{a}_1$ and $\vec{a}_2$ are the in-plane orthogonal unit vectors. The scalar products $(\vec{\sigma}_{\vec{k}}^{\pm} \cdot \vec{\sigma}_{M_h}^{*})$ appearing in Hamiltonian (22) determine the ability of photon circular polarization $\vec{\sigma}_{\vec{k}}^{\pm}$ to create circular polarization $\vec{\sigma}_{M_h}$ with probability $\left|(\vec{\sigma}_{\vec{k}}^{\pm} \cdot \vec{\sigma}_{M_h}^{*})\right|^2$. It can be denoted as a geometrical selection rule. This probability is the same for any in-plane wave vector $k = k_{\parallel}$ and equals to 1/4. If incident wave vector k is perpendicular to layer $k = \vec{a}_3 k_z$, the light with circular polarization $\vec{\sigma}_{\vec{k}}^{\pm}$ excites the magnetoexciton with the same circular polarization $\vec{\sigma}_{M_h} = \vec{\sigma}_{\vec{k}}^{\pm}$ with the probability equal to unity. Another spin–orbital selection rule is determined by coefficients $T\left(R_i, \varepsilon, \vec{k}_{\parallel}\right)$, which are expressed by formulas (36) of [11] as follows:

$$T\left(R_1,\varepsilon_m^-;\vec{k}_{\|}\right)=a_0^{-*}d_{m-3}^{-*}\tilde{\phi}\left(0,m-3;\vec{k}_{\|}\right)-b_1^{-*}c_m^{-*}\tilde{\phi}\left(1,m;\vec{k}_{\|}\right),\ m\geq 3,$$
$$T\left(R_1,\varepsilon_m;\vec{k}_{\|}\right)=-b_1^{-*}\tilde{\phi}\left(1,m;\vec{k}_{\|}\right),\ m=0,1,2,$$
$$T\left(R_2,\varepsilon_m^-;\vec{k}_{\|}\right)=-c_m^{-*}\tilde{\phi}\left(0,m;\vec{k}_{\|}\right),\ m\geq 3,$$
$$T\left(R_2,\varepsilon_m;\vec{k}_{\|}\right)=-\tilde{\phi}\left(0,m;\vec{k}_{\|}\right),\ m=0,1,2 \tag{23}$$

Integrals $\tilde{\phi}\left(n_e,n_h;\vec{k}_{\|}\right)$ were introduces by formulas (32) of [11]. They have a general form and some particular values given below

$$\tilde{\phi}\left(n_e,n_h;\vec{k}_{\|}\right)=\int_{-\infty}^{\infty}dy\varphi_{n_e}\left(y-\frac{k_xl_0^2}{2}\right)\varphi_{n_h}\left(y+\frac{k_xl_0^2}{2}\right)e^{ik_yy},$$
$$\tilde{\phi}\left(0,0;\vec{k}_{\|}\right)=1-\left(\frac{k_x^2}{2}+k_y^2\right)\frac{l_0^2}{4}, \tag{24}$$
$$\tilde{\phi}\left(0,1;\vec{k}_{\|}\right)=\frac{\left(k_x+ik_y\right)l_0}{\sqrt{2}}$$

At point $k_{\|}=0$, they coincide with the normalization and the orthogonality conditions for wave functions $\varphi_n(y)$, which have real values.

Integrals (24) play the role of the orbital selection rules for the quantum transitions from the ground state of the crystal to the magnetoexciton states as well as for the band-to-band optical transitions. Following them in the case of the dipole-active transitions with $k_{\|}=0$, the selection rule is $n_e=n_h$.

The zeroth order Hamiltonian describing the free 2D magnetoexcitons, the cavity photons, and their interaction has a quadratic form and consists of three parts

$$H_2=H_{mex}^0+H_{ph}^0+H_{mex-ph} \tag{25}$$

For simplicity, denotation (18) of the magnetoexciton creation operators will be shortened as follows:

$$\psi_{ex}^+\left(F_1,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_1,-1,\varepsilon_3^-\right)$$
$$\psi_{ex}^+\left(F_2,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_2,-1,\varepsilon_3^-\right)$$
$$\psi_{ex}^+\left(F_3,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_1,1,\varepsilon_0\right)$$
$$\psi_{ex}^+\left(F_4,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_2,1,\varepsilon_0\right) \tag{26}$$
$$\psi_{ex}^+\left(F_5,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_1,-1,\varepsilon_4^-\right)$$
$$\psi_{ex}^+\left(F_6,\vec{k}_{\|}\right)=\psi_{ex}^+\left(\vec{k}_{\|},R_2,-1,\varepsilon_4^-\right)$$

In Hamiltonian H_2, only dipole-active magnetoexciton states F_1 and F_4 and quadrupole-active states F_3 and F_5 were included:

$$H^{0}_{mex} = \sum_{n=1,3,4,5} E_{ex}\left(F_n, \vec{k}_{\parallel}\right)\psi^{+}_{ex}\left(F_n, \vec{k}_{\parallel}\right)\psi_{ex}\left(F_n, \vec{k}_{\parallel}\right) \tag{27}$$

The remaining two states F_2 and F_6 were excluded because they are forbidden in both approximations.

The cavity photons have wave vectors $\vec{k} = a_3 k_z + k_{\parallel}$ consisting of two parts. The longitudinal component is oriented along the axis of the resonator determined by unit vector $\vec{a}_3$ perpendicular to the surface of the QW embedded inside a microcavity. It has a well-defined value of $k_z = \pm\frac{\pi}{L_c}$, where L_c is the cavity length. Transverse component $k_{\parallel} = \vec{a}_1 k_x + \vec{a}_2 k_y$ is a 2D vector oriented in-plane with respect to the QW and determined by two in-plane unit vectors $\vec{a}_1$ and $\vec{a}_2$. Vectors $\vec{\sigma}^{\pm}_{k}$ of the light circular polarizations can be constructed introducing two unit vectors $\vec{s}$ and t perpendicular to light wave vector k and to each other as follows:

$$\begin{aligned} &\vec{\sigma}^{\pm}_{k} = \frac{1}{\sqrt{2}}\left(\vec{s} \pm i\vec{t}\right);\ \vec{k} = \vec{a}_3 k_z + \vec{k}_{\parallel};\ k_z = \pm\frac{\pi}{L_c} \\ &\vec{s} = \vec{a}_3\frac{\left|\vec{k}_{\parallel}\right|}{\left|\vec{k}\right|} - \frac{\vec{k}_{\parallel}\cdot k_z}{\left|\vec{k}\right|\left|\vec{k}_{\parallel}\right|};\ \vec{t} = \frac{\vec{a}_1 k_y - \vec{a}_2 k_x}{\left|\vec{k}_{\parallel}\right|} \end{aligned} \tag{28}$$

They obey the orthogonality and normalization conditions

$$\begin{aligned} &\left(\vec{k}\cdot\vec{t}\right) = \left(\vec{s}\cdot\vec{k}\right) = \left(\vec{t}\cdot\vec{s}\right) = 0;\ \left|\vec{s}\right| = \left|\vec{t}\right| = 1 \\ &\left(\vec{\sigma}^{\pm}_{k}\right)^{*} = \vec{\sigma}^{-}_{k};\ \left|\vec{\sigma}^{\pm}_{k}\right| = 1 \\ &\left(\vec{\sigma}^{\pm}_{k}\cdot\vec{\sigma}^{\mp *}_{k}\right) = 0;\ \left(\vec{\sigma}^{\pm}_{k}\cdot\vec{\sigma}^{\pm *}_{k}\right) = 1 \end{aligned} \tag{29}$$

and have the form

$$\begin{aligned} &\vec{\sigma}^{\pm}_{k} = \frac{1}{\sqrt{2}\left|\vec{k}\right|\left|\vec{k}_{\parallel}\right|}\left\{\vec{a}_3\left|\vec{k}_{\parallel}\right|^2 + \vec{a}_1\left(-k_x k_z \pm ik_y\left|\vec{k}\right|\right) + \vec{a}_2\left(-k_y k_z \mp ik_x\left|\vec{k}\right|\right)\right\}; \\ &\vec{\sigma}_{\pm 1} = \frac{1}{\sqrt{2}}\left(\vec{a}_1 \pm i\vec{a}_2\right) \end{aligned} \tag{30}$$

Here, magnetoexciton circular polarization vectors $\vec{\sigma}_{\pm 1}$ determined by formula (22) are mentioned. The required scalar products of the photon and magnetoexciton circular polarization vectors are listed below:

$$\begin{aligned} &\left|\left(\vec{\sigma}^{\pm}_{k}\cdot\vec{\sigma}^{*}_{1}\right)\right|^2 = \frac{1}{2}\left(1 \pm \frac{k_z}{\left|k_z\right|}\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{2}\right) \mp \frac{x^4}{16}\cdot\frac{k_z}{\left|k_z\right|}, \\ &\left|\left(\vec{\sigma}^{\pm}_{k}\cdot\vec{\sigma}^{*}_{-1}\right)\right|^2 = \frac{1}{2}\left(1 \mp \frac{k_z}{\left|k_z\right|}\right)\left(1 - \frac{x^2}{2} + \frac{x^4}{2}\right) \pm \frac{x^4}{16}\cdot\frac{k_z}{\left|k_z\right|}, \\ &x^2 = \frac{\left|\vec{k}_{\parallel}\right|^2 L_c^2}{\pi^2} < 1. \end{aligned} \tag{31}$$

In the case $k_z = \frac{\pi}{L_c} > 0$, we obtain the expression

$$\begin{aligned} \left|\left(\vec{\sigma}_k^{\pm} \cdot \vec{\sigma}_1^{*}\right)\right|^2 &= \left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 \approx \left(1 - \frac{x^2}{2} + \frac{7}{16}x^4\right), \\ \left|\left(\vec{\sigma}_k^{\pm} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 &= \left|\left\langle\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_1^{*}\right\rangle\right|^2 = \frac{x^4}{16}, \end{aligned} \tag{32}$$

whereas in the opposite case $k_z = -\frac{\pi}{L_c} < 0$ they are

$$\left|\left(\vec{\sigma}_k^{\pm} \cdot \vec{\sigma}_1^{*}\right)\right|^2 = \left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 = \frac{x^4}{16}$$

$$\left|\left(\vec{\sigma}_k^{\pm} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 = \left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_1^{*}\right)\right|^2 \approx \left(1 - \frac{x^2}{2} + \frac{7}{16}x^4\right)$$

The zeroth order Hamiltonian of the cavity photons with wave vectors k and circular polarizations $\vec{\sigma}_k^{\pm}$ described in (28) has the form

$$H_{ph}^{0} = \sum_{\vec{k}_{\|}, k_z = \pm\frac{\pi}{L_C}} \hbar\omega_{\vec{k}} \left[\left(C_{\vec{k},+}\right)^{+} C_{\vec{k},+} + \left(C_{\vec{k},-}\right)^{+} C_{\vec{k},-}\right] \tag{33}$$

The creation and annihilation operators of the photons with circular polarizations $\vec{\sigma}_k^{\pm}$ and with linear polarizations s and t are related as follows:

$$\begin{aligned} C_{\vec{k},\pm} &= \frac{1}{\sqrt{2}}\left(C_{\vec{k},s} \pm iC_{\vec{k},t}\right) \\ \left(C_{\vec{k},\pm}\right)^{+} &= \frac{1}{\sqrt{2}}\left(C_{\vec{k},s}^{+} \mp iC_{\vec{k},t}^{+}\right) \\ \vec{s}C_{\vec{k},s} + \vec{t}C_{\vec{k},t} &= C_{\vec{k},-}\vec{\sigma}_k^{+} + C_{\vec{k},+}\vec{\sigma}_k^{-} \\ \vec{s}C_{\vec{k},s}^{+} + \vec{t}C_{\vec{k},t}^{+} &= \left(C_{\vec{k},-}\right)^{+}\vec{\sigma}_k^{-} + \left(C_{\vec{k},+}\right)^{+}\vec{\sigma}_k^{+} \end{aligned} \tag{34}$$

The Hamiltonian describing the magnetoexciton–photon interaction including only the resonance terms and taking into account the two photon circular polarizations has the form

$$\begin{aligned} H_{mex-ph} = \sum_{\vec{k}_{\|}, k_z = \pm\frac{\pi}{L_c}} &\Big\{\varphi\left(F_1, \vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{+} \cdot \vec{\sigma}_{-1}^{*}\right) + C_{\vec{k},+}\left(\vec{\sigma}_k^{-} \cdot \vec{\sigma}_{-1}^{*}\right)\right]\psi_{ex}^{+}\left(F_1, F_{\|}\right) + \\ &+\varphi\left(F_4, \vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{+} \cdot \vec{\sigma}_1^{*}\right) + C_{\vec{k},+}\left(\vec{\sigma}_k^{-} \cdot \vec{\sigma}_1\right)\right]\psi_{ex}^{+}\left(F_4, \vec{k}_{\|}\right) + \\ &+\varphi\left(F_3, \vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{+} \cdot \vec{\sigma}_1^{*}\right) + C_{\vec{k},+}\left(\vec{\sigma}_k^{-} \cdot \vec{\sigma}_1\right)\right]\psi_{ex}^{+}\left(F_3, \vec{k}_{\|}\right) + \\ &+\varphi\left(F_5, \vec{k}_{\|}\right)\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{+} \cdot \vec{\sigma}_{-1}^{*}\right) + C_{\vec{k},+}\left(\vec{\sigma}_k^{-} \cdot \vec{\sigma}_{-1}^{*}\right)\right]\psi_{ex}^{+}\left(F_5, \vec{k}_{\|}\right) + H.C.\Big\} \end{aligned} \tag{35}$$

Coefficients $\varphi\left(F_n, \vec{k}_{\|}\right)$ and their square moduli are:

$$\varphi\left(F_1,\vec{k}_\parallel\right)=-\phi_{cv}a_0^{-*}d_0^{-*};\quad \left|\varphi\left(F_1,\vec{k}_\parallel\right)\right|^2=\left|\phi_{cv}\right|^2\left|a_0^-\right|^2\left|d_0^-\right|^2$$

$$\varphi\left(F_4,\vec{k}_\parallel\right)=\phi_{cv};\quad \left|\varphi\left(F_4,\vec{k}_\parallel\right)\right|^2=\left|\phi_{cv}\right|^2$$

$$\varphi\left(F_3,\vec{k}_\parallel\right)=\phi_{cv}b_1^{-*}\left(-\frac{k_x+ik_y}{\sqrt{2}}\right)l_0;\quad \left|\varphi\left(F_3,\vec{k}_\parallel\right)\right|^2=\left|\phi_{cv}\right|^2\left|b_1^-\right|^2\frac{\left|\vec{k}_\parallel\right|^2 l_0^2}{2} \tag{36}$$

$$\varphi\left(F_5,\vec{k}_\parallel\right)=-\phi_{cv}a_0^{-*}d_0^{-*}\left(\frac{k_x+ik_y}{\sqrt{2}}\right)l_0;\quad \left|\varphi\left(F_5,\vec{k}_\parallel\right)\right|^2=\left|\phi_{cv}\right|^2\left|a_0^-\right|^2\left|d_0^-\right|^2\cdot\frac{\left|\vec{k}_\parallel\right|^2 l_0^2}{2}$$

$$\phi_{cv}=\frac{e}{m_0 l_0}\sqrt{\frac{\hbar n_c}{\pi c}}P_{cv}(0);\quad \left|\phi_{cv}\right|^2=\left(\frac{\hbar n_c}{\pi c}\right)\left|\frac{eP_{cv}(0)}{m_0 l_0}\right|^2;\quad f_{osc}=\left|\frac{\phi_{cv}}{\hbar\omega_c}\right|^2=\frac{\hbar n_c}{\pi c}\cdot\left|\frac{eP_{cv}(0)}{m_0 l_0\hbar\omega_c}\right|^2$$

Below, the dimensionless value of $f_{osc}=\left|\frac{\phi_{ev}}{\hbar\omega_c}\right|^2$ playing the role of the oscillator strength will be used. Looking at those expressions, one can observe that the probabilities of the dipole-active quantum transitions in magnetoexciton states F_1 and F_4 determined by expressions $\left|\varphi\left(F_1,\vec{k}_\parallel\right)\right|^2$ and $\left|\varphi\left(F_4,\vec{k}_\parallel\right)\right|^2$ are proportional to $\left|\phi_{cv}\right|^2\approx l_0^{-2}\approx B$ and exhibit an increasing linear dependence on magnetic field strength *B*. In contrast, the probabilities of the quadrupole-active quantum transitions in magnetoexciton states F_3 and F_5 are proportional to expression $\left|\phi_{cv}\right|^2\cdot l_0^2$, which does not depend on magnetic field strength B at all.

The equations of motion for four magnetoexciton annihilation operators $\psi_{ex}\left(F_n,\vec{k}_\parallel\right)$ with $n=1,3,4,5$ as well as for the similar photon operators $C_{\vec{k},\pm}$ under the stationary conditions have the form

$$\begin{aligned} &i\hbar\frac{d}{dt}\psi_{ex}\left(F_n,\vec{k}_\parallel\right)=\left[\psi_{ex}\left(F_n,\vec{k}_\parallel\right),\hat{H}_2\right]=\hbar\omega\,\psi_{ex}\left(F_n,\vec{k}_\parallel\right)\\ &i\hbar\frac{d}{dt}C_{\vec{k},\pm}=\left[C_{\vec{k},\pm},\hat{H}_2\right]=\hbar\omega C_{\vec{k},\pm}\end{aligned} \tag{37}$$

Their particular expressions are:

$$\begin{aligned} &\left(\hbar\omega-E_{ex}\left(F_i,\vec{k}_\parallel\right)\right)\psi_{ex}\left(F_i,\vec{k}_\parallel\right)=\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{-1}^{*}\right)+C_{\vec{k},+}\left(\vec{\sigma}_k^{\mp}\cdot\vec{\sigma}_{-1}^{*}\right)\right]\varphi\left(F_i,\vec{k}_\parallel\right)\\ &\qquad i=1,5\\ &\left(\hbar\omega-E_{ex}\left(F_j,\vec{k}_\parallel\right)\right)\psi_{ex}\left(F_j,\vec{k}_\parallel\right)=\left[C_{\vec{k},-}\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{1}^{*}\right)+C_{\vec{k},+}\left(\vec{\sigma}_k^{\mp}\cdot\vec{\sigma}_{1}^{*}\right)\right]\varphi\left(F_j,\vec{k}_\parallel\right)\\ &\qquad j=3,4\\ &\left(\hbar\omega-\hbar\omega_{\vec{k}}\right)C_{\vec{k},\pm}=\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{-1}\right)\varphi^*\left(F_1,\vec{k}_\parallel\right)\psi_{ex}\left(F_1,\vec{k}_\parallel\right)+\\ &+\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{1}\right)\varphi^*\left(F_4,\vec{k}_\parallel\right)\psi\left(F_4,\vec{k}_\parallel\right)+\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{1}\right)\varphi^*\left(F_3,\vec{k}_\parallel\right)\psi_{ex}\left(F_3,\vec{k}_\parallel\right)+\\ &+\left(\vec{\sigma}_k^{\pm}\cdot\vec{\sigma}_{-1}\right)\varphi^*\left(F_5,\vec{k}_\parallel\right)\psi_{ex}\left(F_5,\vec{k}_\parallel\right)\end{aligned} \tag{38}$$

Dipole-active state F_1 and quadrupole-active state F_5 can be preferentially excited by the light with circular polarization $\vec{\sigma}_k^-$ propagating with $k_z = \frac{\pi}{L_c} > 0$, whereas dipole-active state F_4 and the quadrupole-active state F_3 mainly react to the light with circular polarization $\vec{\sigma}_k^+$ propagating in the same direction with $k_z = \frac{\pi}{L_c} > 0$. In the case where Hamiltonians (33) and (35) contain photons only with one circular polarization—either $\vec{\sigma}_k^+$ or $\vec{\sigma}_k^-$— equations (38) give rise to the relations

$$\psi\left(F_i, \vec{k}_\parallel\right) = C_{\vec{k},\pm} \frac{\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{-1}^{*}\right)\varphi\left(F_i, \vec{k}_\parallel\right)}{\hbar\omega - E_{ex}\left(F_i, \vec{k}_\parallel\right)}$$

$$i = 1, 5$$

$$\psi\left(F_j, \vec{k}_\parallel\right) = C_{\vec{k},\pm} \frac{\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{1}^{*}\right)\varphi\left(F_j, \vec{k}_\parallel\right)}{\hbar\omega - E_{ex}\left(F_j, \vec{k}_\parallel\right)} \tag{39}$$

$$j = 3, 4$$

Substituting these relations into equations of motion (38) for photon operators $C_{\vec{k},\pm}$, we will find two dispersion equations describing five branches of the energy spectrum of two different systems. One of them concerns the photons with circular polarization $\vec{\sigma}_k^-$ propagating with $k_z = \frac{\pi}{L_c} > 0$ and exciting mainly magnetoexciton state F_1 as well as other states F_3, F_4, F_5 with smaller oscillator strengths. The second system consists of magnetoexcitons in the same four states F_1, F_3, F_4, F_5 existing in the frame of a microcavity filled by the photons with circular polarization $\vec{\sigma}_k^+$ propagating with $k_z = \frac{\pi}{L_c} > 0$. These fifth order algebraic dispersion equations are

$$\left(\hbar\omega - \hbar\omega_{\vec{k}}\right) = \frac{\left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 \left|\varphi\left(F_1 \vec{k}_\parallel\right)\right|^2}{\hbar\omega - E_{ex}\left(F_1, \vec{k}_\parallel\right)} + \frac{\left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{1}^{*}\right)\right|^2 \left|\varphi\left(F_4, \vec{k}_\parallel\right)\right|^2}{\hbar\omega - E_{ex}\left(F_4, \vec{k}_\parallel\right)} + $$
$$+ \frac{\left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{1}^{*}\right)\right|^2 \left|\varphi\left(F_3 \vec{k}_\parallel\right)\right|^2}{\hbar\omega - E_{ex}\left(F_3, \vec{k}_\parallel\right)} + \frac{\left|\left(\vec{\sigma}_k^{\mp} \cdot \vec{\sigma}_{-1}^{*}\right)\right|^2 \left|\varphi\left(F_5, \vec{k}_\parallel\right)\right|^2}{\hbar\omega - E_{ex}\left(F_5, \vec{k}_\parallel\right)} \tag{40}$$

They describe four magnetoexciton branches and one photon branch with a given circular polarization—either $\vec{\sigma}_k^-$ or $\vec{\sigma}_k^+$—where the photons propagate with $k_z = \frac{\pi}{L_c} > 0$.

Now we will consider a particular case of cavity photons with circular polarization $\vec{\sigma}_k^-$ propagating in the direction with $\vec{k} = \frac{\pi}{L_c} > 0$ in the frame of a microcavity with cavity mode $\hbar\omega_c$

tuned so as to coincide with the magnetoexciton level in state F_1 with wave vector $k_\parallel = 0$ at a given values of magnetic field B, of the electric field E_z, with the parameter of the NP C and with g – factors g_e and g_h. In this case, we can put $\hbar\omega_c = E_{ex}(F_1, B, 0)$. To simplify the dispersion equation, we will use the dimensionless values introduced as follows:

$$\begin{gathered}\hbar\omega = \hbar\omega_c + E \\ \hbar\omega_c = E_{ex}(F_1,B,0)\end{gathered}; \quad \frac{\hbar\omega}{\hbar\omega_c} = 1+\varepsilon; \quad \varepsilon = \frac{E}{\hbar\omega_c} = \frac{E}{E_{ex}(F_1,B,0)} \tag{41}$$

The fifth order dispersion equation has the form

$$\left(\varepsilon - \frac{x^2}{2}\right) = \left(\frac{\hbar n_c}{\pi c}\right)\left|\frac{eP_{cv}}{m_0 l_0 E_{ex}(F_1,B,0)}\right|^2 \times$$
$$\times\left\{\frac{\left|a_0^-\right|^2\left|d_0^-\right|^2\left(1-\frac{x^2}{2}\right)}{\left(\varepsilon - \frac{n_c^2 E_{ex}(F_1,B,0)}{M(F_1,B)c^2}\cdot\frac{x^2}{2}\right)} + \frac{\frac{x^4}{16}}{\left(\varepsilon + 1 - \frac{E_{ex}(F_4,B,0)}{E_{ex}(F_1,B,0)} - \frac{n_c^2 E_{ex}(F_1,B,0)}{M(F_4,B)c^2}\cdot\frac{x^2}{2}\right)} + \right. \tag{42}$$
$$\left. + \frac{\left|b_1^-\right|^2\left(\frac{\pi l_0}{L_c}\right)^2\frac{x^6}{32}}{\left(\varepsilon + 1 - \frac{E_{ex}(F_3,B,0)}{E_{ex}(F_1,B,0)} - \frac{n_c^2 E_{ex}(F_1,B,0)}{M(F_3,B)c^2}\cdot\frac{x^2}{2}\right)} + \frac{\left|a_0^-\right|^2\left|d_0^-\right|^2\left(\frac{\pi l_0}{L_c}\right)^2\frac{x^2}{2}\left(1-\frac{x^2}{2}\right)}{\left(\varepsilon + 1 - \frac{E_{ex}(F_5,B,0)}{E_{ex}(F_1,B,0)}\right) - \frac{n_c^2 E_{ex}(F_1,B,0)}{M(F_5,B)c^2}\cdot\frac{x^2}{2}}\right\}$$

In another special case, the cavity photons have circular polarization $\vec{\sigma}_k^{\pm}$ and propagate in the direction with $\vec{k}_z = \frac{\pi}{L_c} > 0$. Cavity mode energy $\hbar\omega_c$ is tuned to magnetoexciton energy $E_{ex}(F_4, B, 0)$. In this case, the energy is accounted from energy level $E_{ex}(F_4, B, 0)$ as follows:

$$\begin{gathered}\hbar\omega = E_{ex}(F_4,B,0) + E; \quad \frac{\hbar\omega}{E_{ex}(F_4,B,0)} = 1+\varepsilon \\ \varepsilon = \frac{E}{E_{ex}(F_4,B,0)}\end{gathered} \tag{43}$$

The dispersion equation in the dimensionless variables has the form

$$\left(\varepsilon-\frac{x^2}{2}\right)=\left(\frac{\hbar n_c}{\pi c}\right)\left|\frac{eP_{cv}}{m_0 l_0 E_{ex}(F_4,B,0)}\right|^2\times$$

$$\left\{\frac{\left(1-\frac{x^2}{2}\right)}{\left(\varepsilon-\frac{n_c^2 E_{ex}(F_4,B,0)}{M(F_4,B)c^2}\cdot\frac{x^2}{2}\right)}+\frac{\frac{x^4}{16}\left|a_0^-\right|^2\left|d_0^-\right|^2}{\left(\varepsilon+1-\frac{E_{ex}(F_1,B,0)}{E_{ex}(F_4,B,0)}-\frac{n_c^2 E_{ex}(F_4,B,0)}{M(F_1,B)c^2}\cdot\frac{x^2}{2}\right)}+\right. \tag{44}$$

$$\left.+\frac{\left|b_1^-\right|^2\left(\frac{\pi l_0}{L_c}\right)^2\frac{x^2}{2}\left(1-\frac{x^2}{2}\right)}{\left(\varepsilon+1-\frac{E_{ex}(F_3,B,0)}{E_{ex}(F_4,B,0)}-\frac{n_c^2 E_{ex}(F_4,B,0)}{M(F_3,B)c^2}\cdot\frac{x^2}{2}\right)}+\frac{\left|a_0^-\right|^2\left|d_0^-\right|^2\left(\frac{\pi l_0}{L_c}\right)^2\frac{2x^6}{32}}{\left(\varepsilon+1-\frac{E_{ex}(F_5,B,0)}{E_{ex}(F_4,B,0)}-\frac{n_c^2 E_{ex}(F_4,B,0)}{M(F_5,B)c^2}\cdot\frac{x^2}{2}\right)}\right\}$$

Figure 1 shows five dimensionless polariton energy branches as a function of dimensionless wave vector x in the absence of RSOC. They correspond to two different values of magnetic field strength $B = 20 \text{ and } 40\text{ T}$ as well as to two different values of heavy-hole g -factor $g_h = \pm 5$. The electron g -factor is assumed to be $g_e = 1$. Figures 1a and 1b suggest that a change in magnetic field strength B does not have a considerable effect on the resulting pattern. Nevertheless, it should be borne in mind that the dimensionless values of $E_p(x)/\hbar\omega_c$ were calculated with different values of cavity mode $\hbar\omega_c = E_{ex}(F_4,B,0)$ depending on B . Figures 2 and 3 show the influence of the RSOC with third order chirality terms and NP parameter *C*. The main effect consists in the transposition of the magnetoexciton energy levels on the energy scale in comparison with their position in the absence of RSOC.

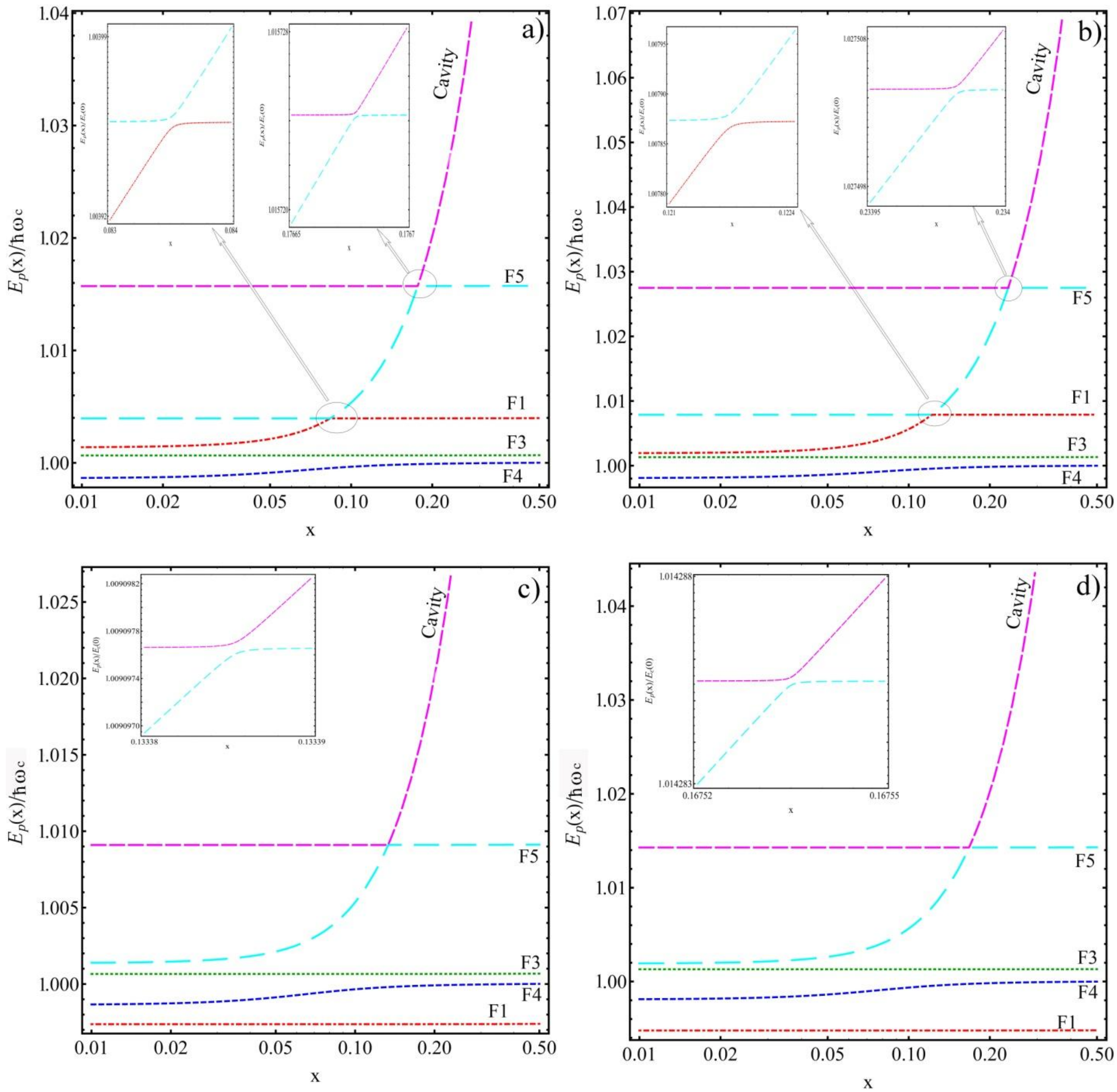

Fig. 1. Dimensionless polariton energy branches as a function of dimensionless wave vector x in the absence of RSOC $(E_z = 0;\ C = 0)$ at different values of magnetic field strength *B*, two values of the heavy-hole $g-\text{factor}\ g_h = \pm 5$ and at a given value of the electron g - factor $g_e = 1$, as follows: $a)\ B = 20\ \text{T},\ g_h = 5;$ $b)\ B = 40\ \text{T},\ g_h = 5;$ $c)\ B = 20\ \text{T},\ g_h = -5;$ $d)\ B = 40\ \text{T},\ g_h = -5$. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity $\hbar\omega_c = E_{ex}(F_4, B, 0)$.

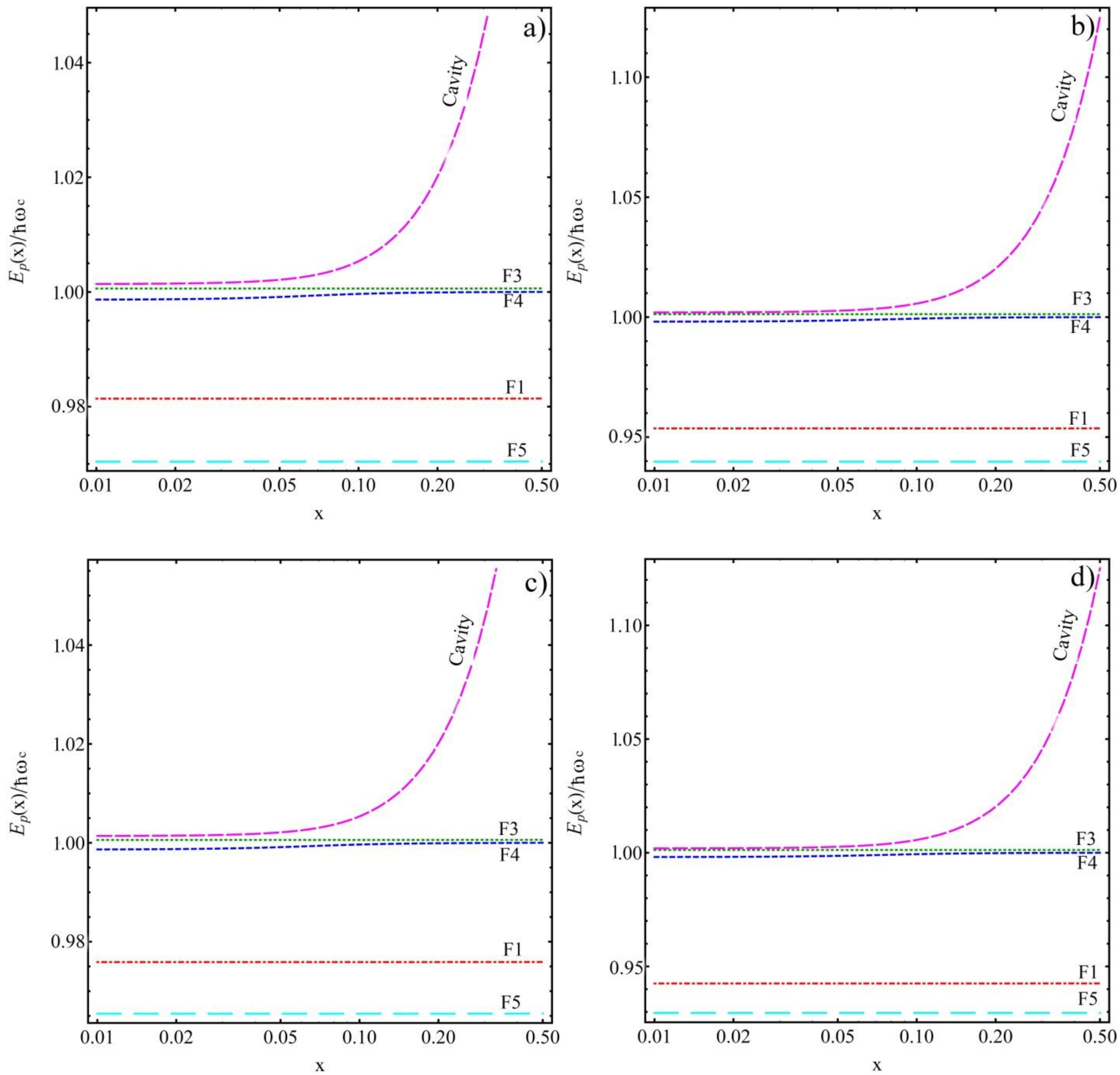

Fig. 2. Dimensionless polariton energy branches as a function of dimensionless wave vector x in the presence of the *RSOC* with electric field strength $E_z = 30 \frac{\mathrm{kV}}{\mathrm{cm}}$ and the parameter of the NP $C = 20$ at different values of magnetic field strength B, two values of the heavy-hole $g-\mathrm{factor}\ g_h = \pm 5$ and electron $g-\mathrm{factor}\ g_e = 1$, as follows: $a)\ B = 20\ \mathrm{T},\ g_h = 5;$ $b)\ B = 40\ \mathrm{T},\ g_h = 5;$ $c)\ B = 20\ \mathrm{T},\ g_h = -5;$ $d)\ B = 40\ \mathrm{T},\ \mathrm{and}\ g_h = -5$. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity. $\hbar\omega_c = E_{ex}(F_4, B, 0)$.

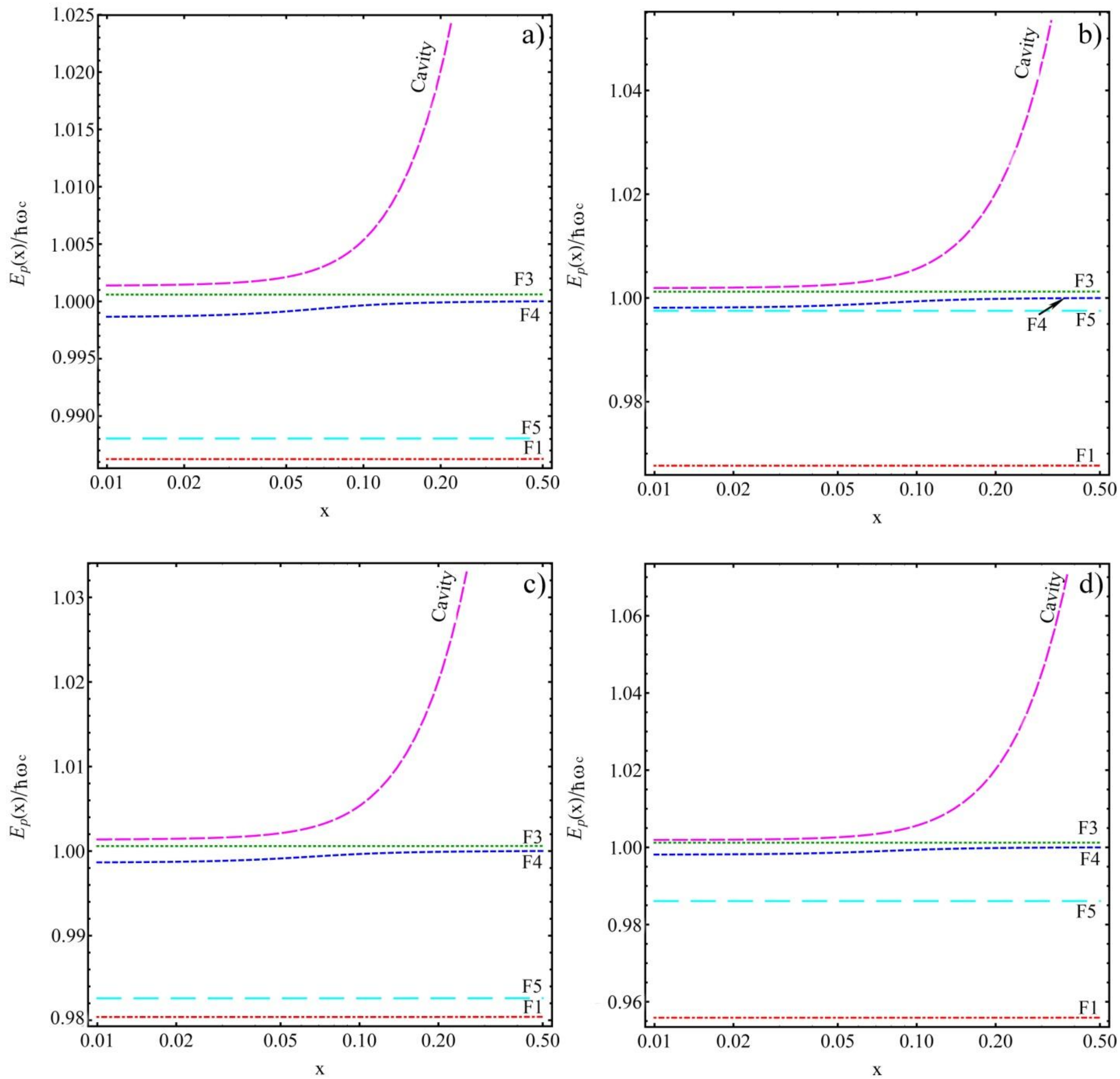

Fig. 3. Dimensionless polariton energy branches as a function of dimensionless wave vector x in the presence of the *RSOC* with electric field strength $E_z = 30\frac{\text{kV}}{\text{cm}}$ and the parameter of the NP $C = 30$ at different values of magnetic field strength B, two values of the heavy-hole $g-\text{factor}\, g_h = \pm 5$ and electron $g-\text{factor}\, g_e = 1$, as follows: $a)\, B = 20\ \text{T},\ g_h = 5;$ $b)\, B = 40\ \text{T},\ g_h = 5;$ $c)\, B = 20\ \text{T},\ g_h = -5;$ $d)\, B = 40\ \text{T},\ \text{and}\ g_h = -5$. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity. $\hbar\omega_c = E_{ex}(F_4, B, 0)$

The following three figures show the case where the cavity mode is tuned to magnetoexciton energy level $E_{ex}(F_1, B, 0)$ rather than to level $E_{ex}(F_4, B, 0)$ as was supposed in the previous three ones. The three variants at a given energy level $E_{ex}(F_1, B, 0)$ are related with the absence (Fig. 4) and presence (Figs. 5, 6) of the RSOC with two different values of NP constant *C = 20* and *30*.

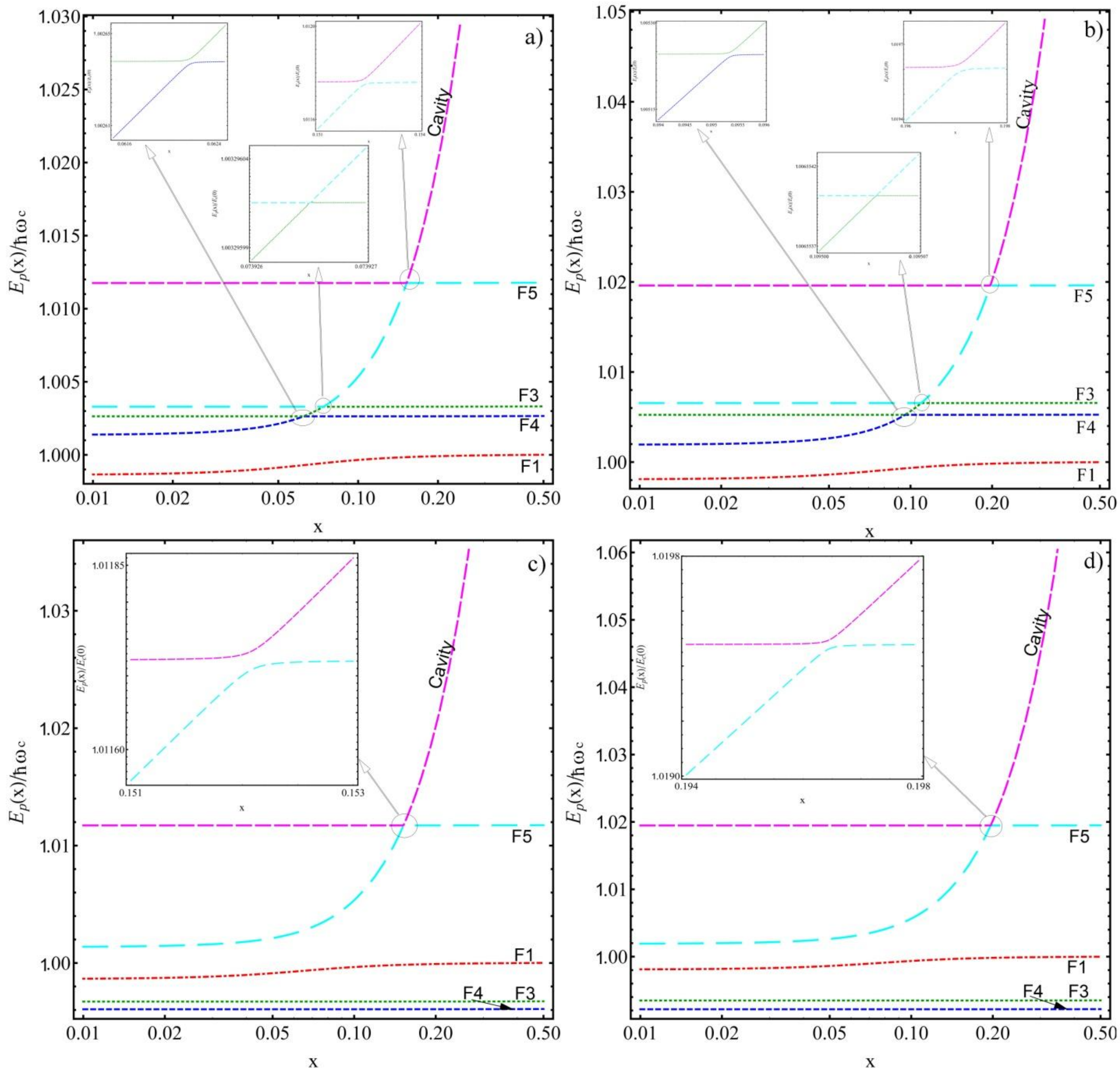

Fig. 4. Dimensionless polariton energy branches in circular polarization $\vec{\sigma}^-$ as a function of dimensionless wave vector *x* in the absence of the RSOC ($E_z = 0$, $C = 0$) at the two different values of the magnetic field strength, at two values of the heavy-hole g-factor $g_h = \pm 5$ and at a given value of the electron g-factor $g_e = 1$ as follows: (a) *B* = 20 T, g_h = 5; (b) *B* = 40 T, g_h = 5; (c) *B* = 20 T, g_h = -5; (d) *B* = 40 T, g_h = -5. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity. $\hbar\omega_c = E_{ex}(F_1, B, 0)$.

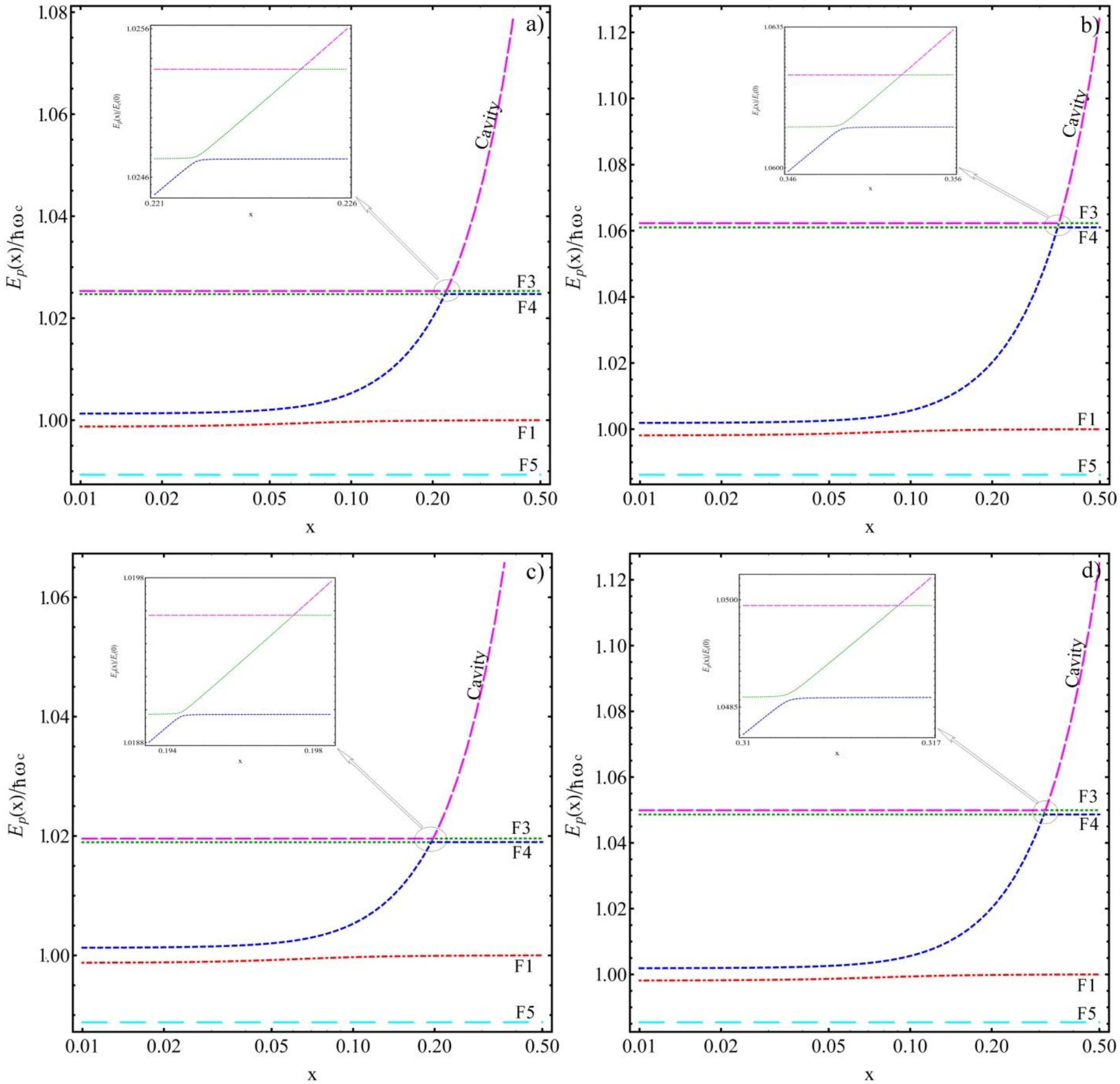

Fig. 5. Dimensionless polariton energy in circular polarization $\vec{\sigma}^-$ as a function of dimensionless wave vector *x* in the presence of the RSOC with electric field strength $E_z = 30\,kV/cm$ and the parameter of NP $C = 20$ at different values of magnetic field strength *B*, at two values of the heavy-hole g-factor $g_h = \pm 5$ and at the electron g-actor $g_e = 1$ as follows: (a) $B = 20$ T, $g_h = 5$; (b) $B = 40$ T, $g_h = 5$; (c) $B = 20$ T, $g_h = -5$; (d) $B = 40$ T, $g_h = -5$. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity. $\hbar\omega_c = E_{ex}(F_1, B, 0)$.

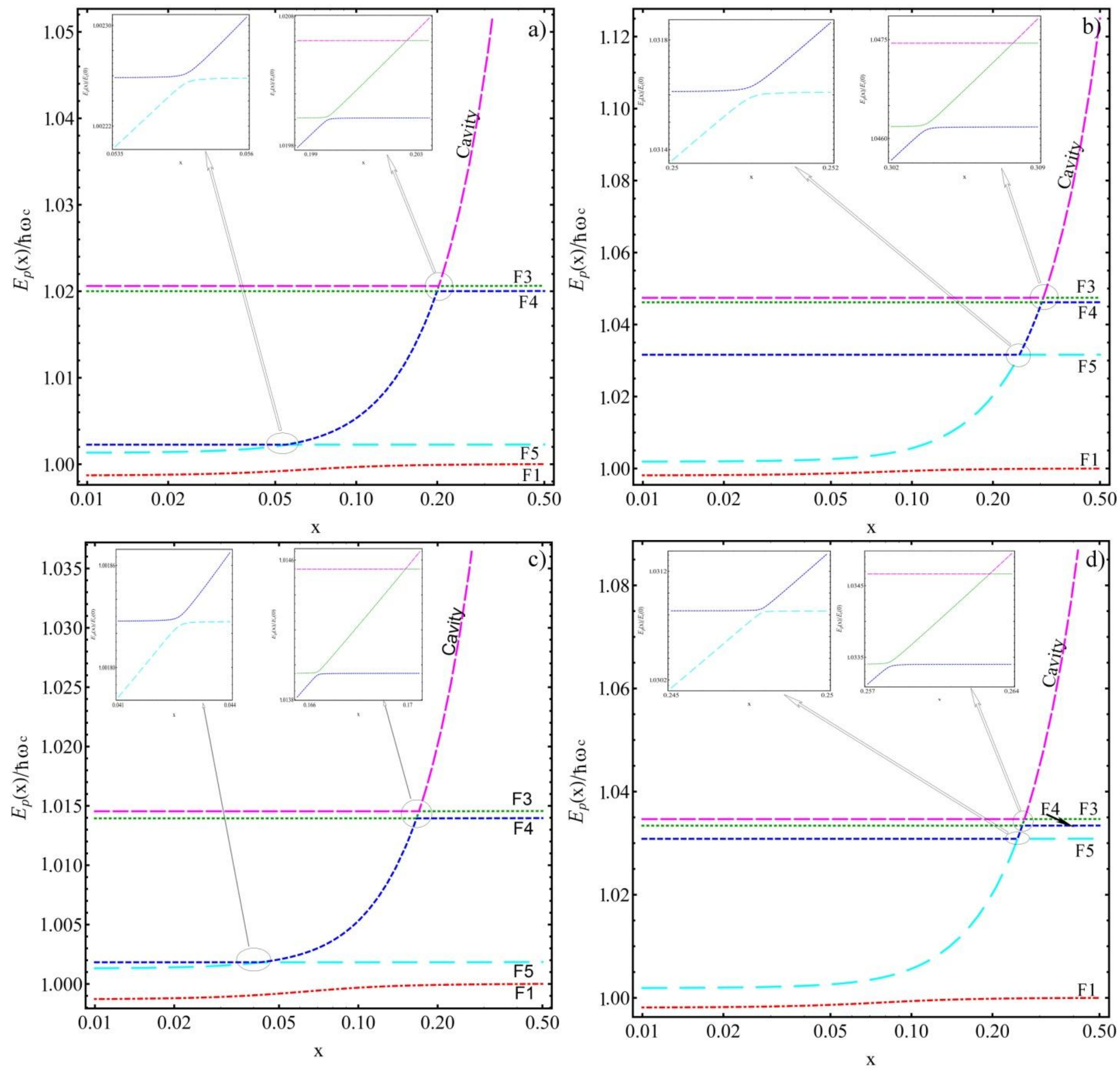

Fig. 6. Dimensionless polariton energy in circular polarization $\vec{\sigma}^-$ as a function of dimensionless wave vector *x* in the presence of the RSOC with electric field strength $E_z = 30\,\mathrm{kV/cm}$ and the parameter of NP $C = 30$ at different values of magnetic field strength *B*, at two values of the heavy-hole g-factor $g_h = \pm 5$ and at the electron g-actor $g_e = 1$ as follows: (a) $B = 20$ T, $g_h = 5$; (b) $B = 40$ T, $g_h = 5$; (c) $B = 20$ T, $g_h = -5$; (d) $B = 40$ T, $g_h = -5$. The magnetoexciton energy levels are denoted by F_1, F_3, F_4, F_5, whereas the cavity mode by Cavity. $\hbar\omega_c = E_{ex}(F_1, B, 0)$.

5. Conclusions

The properties of the 2D cavity polaritons subjected to the action of strong perpendicular magnetic and electric fields have been studied. To this end, the exact solutions of the LQ of the 2D heavy-holes accompanied by the RSOC with third order chirality terms, by the ZS effects as well as by the nonparabolicity of their dispersion law have been obtained following the method proposed by Rashba [1]. His results concerning the conduction electrons have been supplemented taking into account the ZS effects. Using the mentioned wave functions for the 2D electrons and holes, the Hamiltonians describing the Coulomb electron-electron and the electron-radiation interactions in the second quantization representation have been deduced. In turn, the electron-radiation interaction Hamiltonian has made it possible to construct another Hamiltonian describing the magnetoexciton–photon interaction and begin the development of the theory of magnetoexciton-polaritons. To do this, the wave functions of the 2D magnetoexcitons are required. The six magnetoexciton states arising due to the composition of two LLLs for conduction electrons with three LLLs for heavy-holes have been taken into consideration. Between them, two states—F_1 and F_4—are dipole-active, other two—F_3 and F_5—are quadrupole-active, and the last two—F_2 and F_6—are forbidden in the inter-band optical quantum transitions as well as from the ground state of the crystal to the magnetoexciton states in the GaAs-type QWs. The dispersion equation describing the magnetoexciton-polariton energy spectrum includes the first four states F_1, F_3, F_4 and F_5 interacting with the cavity photons in two selections of the cavity mode. These four magnetoexciton degrees of freedom together with the branch of the cavity photons give rise to five order dispersion equation with five polariton-type renormalized energy branches. They are represented in six figures. Three of them show the case of cavity mode energy $\hbar\omega_c$ tuned exactly to the magnetoexciton dipole-active level energy $E_{ex}(F_4, B, 0)$, whereas the other three figure show the polariton pictures where the cavity mode energy is tuned exactly to the other dipole-active magnetoexciton level energy $E_{ex}(F_1, B, 0)$. In both cases, a multitude of the polariton energy branches has been obtained.

Acknowledgments. E. V. D. and I.V.P. thank the Foundation for Young Scientists of the Academy of Sciences of Moldova for financial support (14.819.02.18F).

References:

[1] E. I. Rashba, Sov. Phys. Solid. State, 2, 1109, (1960)
[2] S. A. Moskalenko, I. V. Podlesny, P. I. Khadzhi, B. V. Novikov, and A. A. Kiselyov, Solid State Commun., 151, 1690, (2011)
[3] T. Hakioglu, M. A. Liberman, S. A. Moskalenko, and I. V. Podlesny, J. Phys.: Cond. Matt., 23, 345405, (2011)
[4] I. V. Podlesny, S. A. Moskalenko, T. Hakioglu, A. A. Kiselyov, and L. E. Gherciu, Physica E, 49, 44, (2013)
[5] S. A. Moskalenko, I. V. Podlesny, E. V. Dumanov, and A. A. Kiselyov, J. Nanoelectron. Optoelectron., 9, 219, (2014)
[6] S. A. Moskalenko, I. V. Podlesny, E. V. Dumanov, M. A. Liberman, and B. V. Novikov, Physica Status Solidi B (in press)
[7] R. Winkler, Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems, Berlin, Springer Tracts in Modern Physics, vol. 191, (2003)
[8] T. Hakioglu and M. S. Sahin, Phys. Rev. Lett., 98, 166405 (2007)

[9] S. A. Moskalenko, M. A. Liberman, and I. V. Podlesny, Phys. Rev. B, 79(12), 125425, (2009)

[10] S. A. Moskalenko, I. V. Podlesny, M. A. Liberman, and B. V. Novikov, J. Nanophoton. 6, 061806, (2012)

[11] S. A. Moskalenko, I. V. Podlesny, E. V. Dumanov, and A. A. Kiselyov, J. Nanoelectron. Optoelectron. 9(2), 219, (2014)

[12] S. A. Moskalenko, E. V. Dumanov, I. V. Podlesny, M. A. Liberman, B. V. Novikov, S. S. Rusu, and V. M. Bajireanu, Mold. J. Phys. Sc. 13(1-2), (2014)

[13] S. A. Moskalenko, I. V. Podlesny, E. V. Dumanov, and A. A. Kiselyov, J. Nanoelectron. Optoelectron. (in press)