ELABORATION AND RESEARCH OF PLANETARY PRECESSIONAL MULTIPLIER

R. Ciobanu, PhD student
Technical University of Moldova

1. INTRODUCTION

The multiplier is an indispensable part of the micro hydropower plant and high power wind turbine. It helps to increase rotor low speeds limited by the water flow small velocity and by the relative big placement diameter of the blades that participate in the energy conversion. For example, the microhydrostation rotor's speed is $(2 - 3)\ min^{-1}$ for water flow velocity $V=(1...1,6)\ m/s$ and for blade placement diameter $D = 4\ m$.

Diversity of requirements forwarded by the beneficiaries of mechanical transmissions consists, in particular, in increasing reliability, efficiency and lifting capacity, and in reducing the mass and dimensions. It becomes more and more difficult to satisfy the mentioned demands by partial updating of traditional transmissions. The target problem can be solved with special effects by developing new types of multipliers based on precessional planetary transmissions with multiple gear, that were developed by the authors. Absolute multiplicity of precessional gear (up to *100%* pairs of teeth simultaneously involved in gearing, compared to *5%-7%* - in classical gearings) provides increased lifting capacity and small mass and dimensions. To mention that until now precessional planetary transmissions have been researched and applied mainly in reducers. Therefore it was necessary to carry out theoretical research to determine the geometrical parameters of the precessional gear that operates in multiplier mode. Also, it was necessary to develop new conceptual diagrams of precessional transmissions that function under multiplier regime.

The majority of precessional planetary transmissions diagrams developed previously operate efficiently in reducer's regime [1]. Depending on the structural diagram, precessional transmissions fall into two main types – *K-H-V* and *2K-H*, from which a wide range of constructive solutions with wide kinematical and functional options that operate in multiplier regime. The kinematical diagram of the precessional transmission *K-H-V* (fig. 1), comprises five basic elements: planet career *H*, satellite gear *g*, two central wheels b with the same number of teeth, controlling mechanism *W* and the body (frame). The

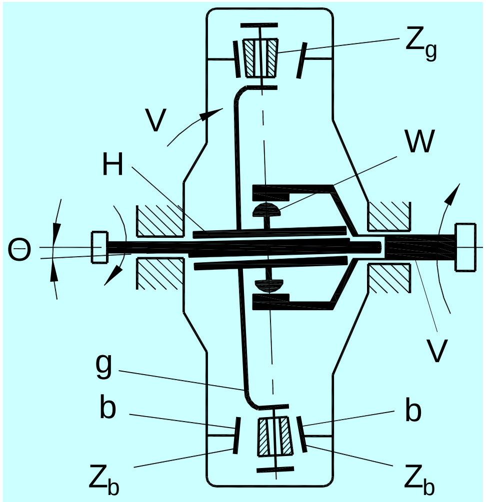


Figure 1. Conceptual diagrams of precesional transmissions that operates efficiently in the multiplication regime.

roller rim of the satellite gear g gears internally with the sun wheels b, and their teeth generators cross in a point, so-called the centre of precession. The satellite gear g is mounted on the planet (wheel) career *H*, designed in the form of a sloped crank, which axis forms some angle with the central wheel axis θ.

Revolving, the sloped crank *H* transmits sphero-spatial motion to the satellite wheel regarding the ball hinge installed in the centre of precession. For the transmission with the controlling mechanism designed as clutch coupling (fig.1), the gear ratio (gear reduction rate) varies in the limits:

$$i_{HV}^{g} = -\frac{z_g \cos\Theta - z_b}{z_b};\quad i_{HV}^{g} = -\frac{z_g \cos\Theta - z_b}{z_b \cos\Theta}, \qquad (1)$$

reaching the extreme values of *4* times for each revolution of the crank *H*. If necessary this shortcoming can be eliminated using as a controlling mechanism the constant cardan joint (Hooke's joint), the ball synchronous couplings, etc.

$$i_{HVmed}^{g} = -\frac{z_g - z_b}{z_b}. \qquad (2)$$

$$i_{HV}^{g} = -\frac{1}{z_b},$$

For $z_g = z_b+1$, the driving and driven shafts have opposite directions.

$$i_{HV}^{g} = \frac{1}{z_b},$$

For $z_g = z_b-1$, the shafts revolve in the same direction.

This kinematical diagram of the precessional transmission ensures a range of gear ratios $i = 8...60$, but in the multiplication regime it operates efficiently only for the range of gear ratios $i = 8...25$. As well, in the coupling mechanism *W*, that operates with pitch angles of the semi couplings up to 3^o, power losses occur reducing the efficiency of the multiplier on the whole.

2. ANALYTIC DESCRIPTION OF TEETH PROFILE AND JUSTIFICATION OF PRECESSIONAL GEAR PARAMETERS SELECTION

Teeth profiles have an important role in the efficient transformation of motion in the precessional transmissions that operate as multiplier. Multiple precessional gear theory, previously developed, did not take into consideration the influence of the diagram error of the linking mechanism in the processing device for gear wheel on the teeth profile. Functioning under the multiplication regime, these errors have major influence, which can lead to instant blocking of gear and to power losses. With this purpose, a thorough analysis was conducted on the motion development mechanism under multiplication, and on the teeth profile error generating source. On the basis of fundamental theory of multiple precessional gear, previously developed, a new gear with modified teeth profile and the technology for its industrial manufacturing was proposed and patented [2].

Kinematically, the link between the semi product and the tool, in which one of them (the tool) makes spherical-spatial motion being, at the same time, limited from rotating around the axis of the main shaft of the teething machine tool, is similar to the *„satellite-driven shaft”* link from the precessional planetary transmission of the *K-H-V* type. The kinematical link between the tool and the stationary part of the device represents a Hooke articulation that generates the variability of transfer function in the kinematical link *„tool-semi product”*. This variation will influence the teeth profile. Thus, the connection of tool with the housing registers a certain diagram error $\Delta\psi_3$ (to understand the deviation of the semi product angle of rotation ψ_3 from the angle of rotation of the semi product itself ψ_3^m at its uniform rotation):

$$u_{31}^{m} = -\frac{z_2 - z_3}{z_3}; \Delta\psi_3 = \psi_3 - u_{31}^{m} = \frac{z_2}{z_3}\left(\psi - arctg(\cos\theta \cdot tg\psi)\right). \quad (3)$$

Fig. 2 show the dependence of the tool position diagram error $\Delta\psi_3$ at a revolution of the machine tool main shaft ψ. This error is transmitted to the tool that shapes the teeth profile with the

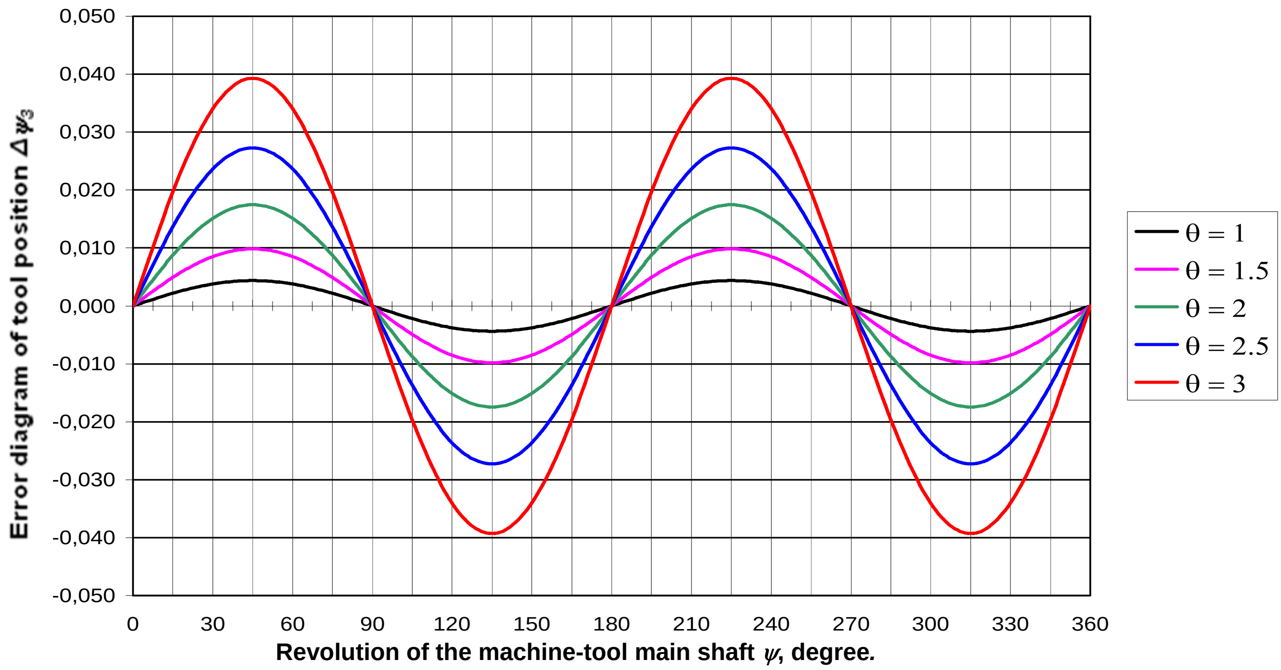


Figure 2. Dependence of the error diagram of tool position $\Delta\psi_3$ at a revolution of the machine-tool main shaft ψ.

same error. To ensure continuity of the transfer function and to improve the performances of precessional transmission under multiplication it is necessary to modify teeth profile with the diagram error value $\Delta\psi_3$ by communicating supplementary motion to the tool. In this case the momentary transmission ratio of the manufactured gear will be constant. Usually, in theoretical mechanics the position of the body making spherical-spatial motion is described by Euler angles. The mobile coordinate system *OX1Y1Z1* is connected rigidly with the satellite wheel, which origin coincides with the centre of precession *0* (Fig. 3) and performs spherical-spatial motion together with the satellite wheel relative to the motionless coordinate system *OXYZ*.

The elaboration of the mathematic model of the modified teeth profile is based integrally on the mathematic model of teeth profile, previously developed by the authors. With this purpose it is necessary to present the detailed description of teeth profile without modification and, then, to present of the description of modified profile peculiarities.

Description of teeth profile designed on sphere. An arbitrary point *D* of the tool axis describes a trajectory relative to the fixed system according to the equations:

$$X_D^m = -\sin\delta\sin\left[Y_C^m\sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right];$$
$$Y_D^m = -Y_C^m\cos\delta + Z_C^m\sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right];$$
$$Z_D^m = -Y_C^m\sin\delta\left(\cos^2\psi + \cos\theta\sin^2\psi\right) - Z_C^m\cos\delta. \quad (4)$$

Index *m* means *„modified”*. The motion of point Dm compared to the movable system connected rigidly to the semi product is described by formulas:

$$X_{1D}^m = X_D^m\cos\frac{\psi}{Z_1} - Y_D^m\sin\frac{\psi}{Z_1};$$
$$Y_{1D}^m = X_D^m\sin\frac{\psi}{Z_1} + Y_D^m\cos\frac{\psi}{Z_1};$$
$$Z_{1D}^m = Z_D^m. \quad (5)$$

The projections of point Dm velocities *OXYZ* and *OX1Y1Z1* is expressed by formulas:

$$\dot{X}_D^m = -\sin\delta\cos\psi\left[Y_C^m\sin\theta + Z_C^m\left(1-\cos\theta\right)\cos\psi\right]\dot{\psi} -$$
$$-\sin\delta\sin\psi\left[\dot{Y}_C^m\sin\theta + \dot{Z}_C^m\left(1-\cos\theta\right)\cos\psi - Z_C^m\left(1-\cos\theta\right)\sin\psi\cdot\dot{\psi}\right];$$
$$\dot{Y}_D^m = -\dot{Y}_C^m\cos\delta + \dot{Z}_C^m\sin\delta\left[\cos^2\psi + \cos\theta\sin^2\psi\right] +$$
$$+Z_C^m\sin\delta\left[-2\cos\psi\sin\psi + 2\cos\theta\sin\psi\cos\psi\right]\dot{\psi};$$

$$\dot{X}_{1D}^m = \dot{X}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}X_D^m\sin\frac{\psi}{Z_1} - \dot{Y}_D^m\sin\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\cos\frac{\psi}{Z_1};$$
$$\dot{Y}_{1D}^m = \dot{X}_D^m\sin\frac{\psi}{Z_1} + \frac{\dot{\psi}}{Z_1}X_D^m\cos\frac{\psi}{Z_1} + \dot{Y}_D^m\cos\frac{\psi}{Z_1} - \frac{\dot{\psi}}{Z_1}Y_D^m\sin\frac{\psi}{Z_1}. \quad (6)$$

The coordinates of point Em on the sphere is calculated by formulas:

$$X_{1E}^m = k_2^m Z_{1E}^m + d_2^m;$$
$$Y_{1E}^m = k_1^m Z_{1E}^m - d_1^m;$$
$$Z_{1E}^m = \frac{(k_1^m d_1^m - k_2^m d_2^m) - \sqrt{(k_1^m d_1^m - k_2^m d_2^m)^2 + (k_1^{m2} + k_2^{m2} + 1)\cdot(R_D^2 - d_1^{m2} - d_2^{m2})}}{k_1^{m2} + k_2^{m2} + 1}, \quad (7)$$

where:

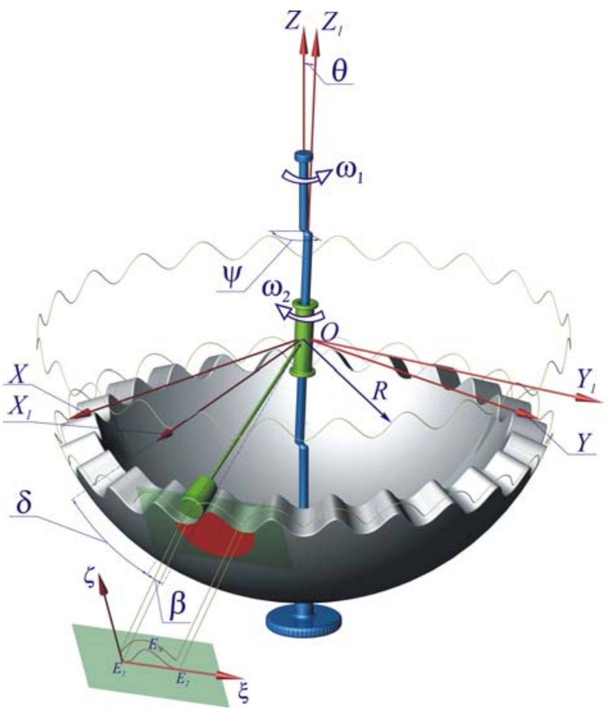


Figure 3. Tooth profile in normal section.

$$k_1^m = \frac{X_{1D}^m\left(X_{1D}^m\dot{X}_{1D}^m + Y_{1D}^m\dot{Y}_{1D}^m\right) + Z_{1D}^{m\,2}\dot{X}_{1D}^m}{Z_{1D}^m\left(X_{1D}^m\dot{Y}_{1D}^m - Y_{1D}^m\dot{X}_{1D}^m\right)}; \quad k_2^m = -\frac{\left(k_1^m Y_{1D}^m + Z_{1D}^m\right)}{X_{1D}^m};$$

$$d_1^m = \frac{R_D^2\cos\beta\,\dot{X}_{1D}^m}{\left(X_{1D}^m\dot{Y}_{1D}^m - \dot{X}_{1D}^m Y_{1D}^m\right)}; \quad d_2^m = \frac{\left(R_D^2\cos\beta + d_1^m Y_{1D}^m\right)}{X_{1D}^m}.$$

According to the obtained analytical relations a soft for the calculation and generation of teeth was developed in CATIA V5R7 modelling system that allowed obtaining the modified trajectories of points Em_e and Em_i on the spherical front surfaces, both exterior and interior ones, by which the teeth surface was generated (Fig. 4).

Description of modified teeth profile projected on a transversal surface. Projection of point E_m on the tooth transversal plane has the following coordinates:

$$X_E^{\prime\prime m} = \varepsilon^m \cdot X_{IE}^m, \quad Y_E^{\prime\prime m} = \varepsilon^m \cdot Y_{IE}^m, \quad Z_E^{\prime\prime m} = \varepsilon^m \cdot Z_{IE}^m, \quad (8)$$

where $$\varepsilon^m = -\frac{D}{AX_{IE}^m + BY_{IE}^m + CZ_{IE}^m}.$$

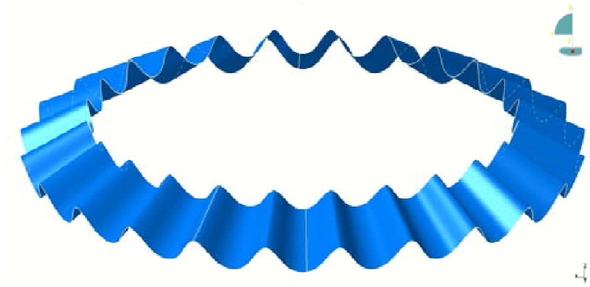


Figure 4. Teeth generating surface.

The modified teeth profile in plane is described by the equations:

$$\xi^m = X_E^{\prime\prime m} \cos\frac{\pi}{Z_1} + \left[R_D \cos(\delta + \theta + \beta) + Y_E^{\prime\prime m}\right] \sin\frac{\pi}{Z_1};$$

$$\zeta^m = X_E^{\prime\prime m} \sin\gamma \sin\frac{\pi}{Z_1} - \left[R_D \cos(\delta + \theta + \beta) + Y_E^{\prime\prime m}\right] \sin\gamma \cos\frac{\pi}{Z_1} +$$

$$+ \left[R_D \sin(\delta + \theta + \beta) + Z_E^{\prime\prime m}\right] \cos\gamma. \quad (9)$$

A wide range of modified teeth profiles with different geometrical parameters were generated in MathCAD 2001 Professional software (Fig. 5 a,b). The solid model of a gear wheel is shown in Fig. 6. Based on the carried out research it was established that from the point of view of decreasing energy losses in gearing, in the multiplication mode of operation, the gearing angle should be $\alpha > 450$, and the nutation angle (the pitch angle of the crank shaft) should be– $\theta \leq 2,50$.

This is dictated by the reverse principle of movement in the multipliers compared to the reducers: the axial component of the normal force in gear must be maximal to drive the crank shaft in the rotation movement through the satellite wheel.

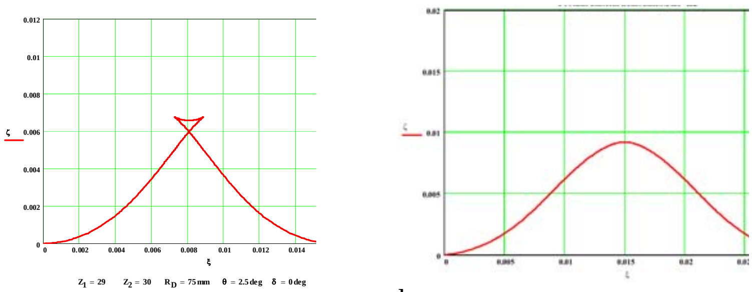
3. ELABORATION OF PLANETARY PRECESSIONAL MULTIPLIER

To avoid power losses in the multiplier and to widen the kinematical options, (Fig. 7) [4] the conceptual diagram of the precessional multiplier with wide kinematical options was designed. The planetary precessional multiplier comprises the following units: the housing *1*, inside which the fixed sun wheel *2* is placed and connected rigidly to the housing cover *3*, exterior satellite wheel *4* with the teeth in the shape of rollers, movable sun wheel *5*, linked rigidly to the input shaft *6*. The satellite wheel *3* is connected kinematically with the sloped flange of the disk *7*, connected rigidly with the sun wheel *8*, that gears with the interior satellite wheel *9* mounted unbound on the output crank shaft *10*, and linked rigidly to the rotor generator *11*. The exterior satellite wheel *4* is mounted void on gear bodies on exterior spherical surfaces of the interior wheel *9*. The pitch angle of the output crank shaft *10* axis and of the sloped flange is equal to θ. The rotational motion of the input shaft 6 is transmitted to the movable sun wheel *5*. Due to the difference in the number of teeth of wheel *5* and of the exterior satellite wheel (gear) *4* *(Z6 = Z5±1)*, the last will have to carry out a precession motion around the fixed point O (centre of precession). Precessional motion around its axis is excluded as the number of teeth of the sun wheel *2* is equal to the number of rollers of the satellite gear *4* *(Z2 = Z4)*. The precessional motion of the exterior satellite gear *4* is transformed, by means of an inclined flange of the disc *7*, into rotational motion around the disc axis *7* that will revolve by the degree of multiplication

$$i_7 = -\frac{z_5}{z_4 - z_5}, \quad (10)$$

where *Z4* is the number of rollers of the exterior satellite gear *4*;

Z5 – is the number of teeth of the movable sun wheel *5*.



a. b.

Figure 5. Teeth profiles for multipliers.

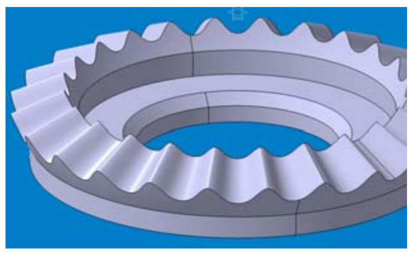


Figure 6. Computerised model of the sun gear.

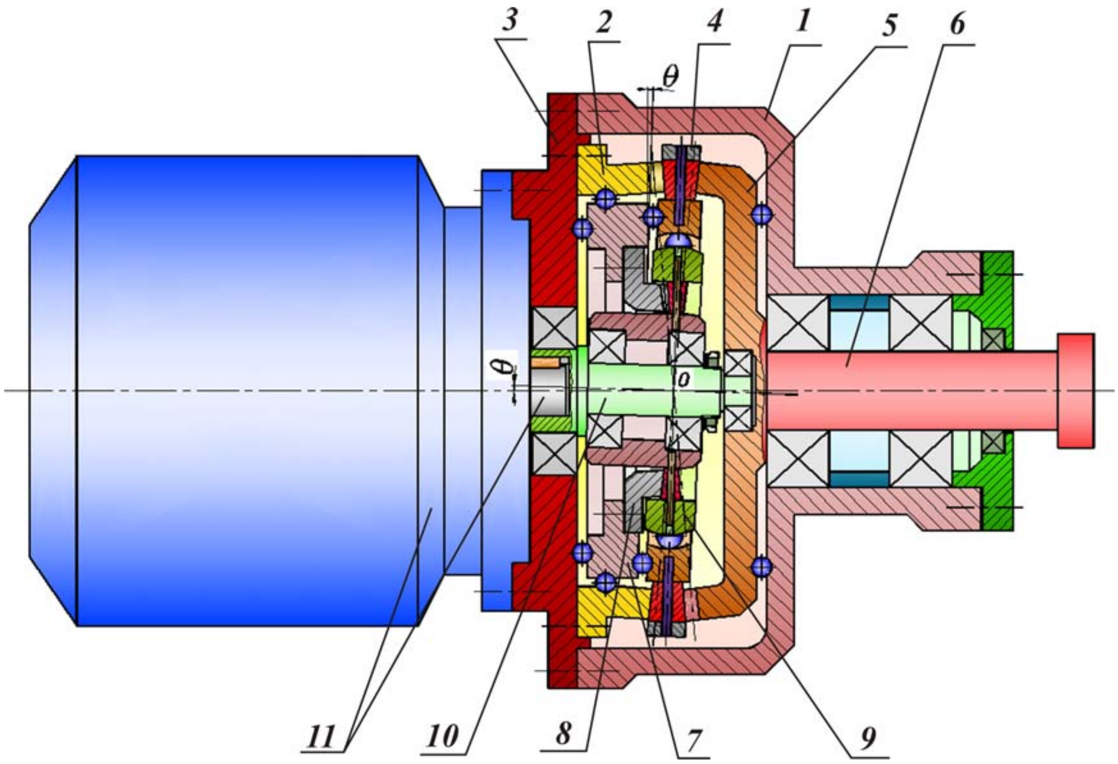


Fig. 7. Planetary precessional multiplier with satellite gear mounted radially.

The multiplied rotational motion of the disc *7* is transformed into multiplied precessional motion of the interior satellite gear *9*, due to the difference in the number of teeth *Z8=Z9±1*. At the rotation of disc *7* of the sun wheel *8* at an angle equal to the teeth angular pitch, the interior satellite gear will perform a complete precession cycle around point *„O”*. The precessional motion of the interior satellite gear *9* is transformed by means of output crank shaft *10* into rotational motion, multiplied to the degree of multiplication

$$i_{10} = -\frac{z_8}{z_9 - z_8}, \qquad (11)$$

where *Z8* is the number of teeth of the sun wheel *8*,

Z9 – is the number of rollers of the satellite gear *9*.

The multiplied rotational motion of the output crank shaft *10* is transmitted to the electrical generator rotor *11*.

CONCLUSIONS

It is necessary to point out the series of peculiarities of the planetary precessional transmissions that ensure higher performances compared to similar planetary transmissions with cylindrical gears: precessional transmissions do not demand conditions of distance equality between the axis. This factor widens the area of their optimal design; precessional transmission kinematics does not limit the selection of the gear couples modules or of the rollers placement pitch. This factor increases the possibilities of shaping teeth pairs and of the transmission ratios interval; the peculiarities of the designed precessional gears allow increasing in the number of teeth that transmit the load simultaneously and this fact reduces significantly the dimensions and mass for the same loads compared to the traditional involute gearings.

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