

# Optimal placement of synchronized phasor measurement technologies based on different methods of optimization

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**Abstract**— This paper presents an analysis and systematization of the diverse methods on the optimal placement of the phasor measurement units (PMU). It is proposed the classification of the methods developed up to date in different articles, there are highlighted the advantages and disadvantages of these methods, considering the variety of the cases of their application. There is analyzed the novel method of determinant measures for optimal PMU placement.

**Keywords**—PMU; optimal placement; observability; data redundancy; contingencies.

## I. INTRODUCTION

Safe and reliable operation of power systems requires the continuous monitoring of operating conditions in a system. Thus the problem of application phasor synchronized measurements becomes actual for power system estimation and ensures a lot of advantages. The main element of phasor synchronized measurements also called intelligent measurements is Phasor Measurement Unit (PMU). PMU provides direct sequence current and voltage measurements with high accuracy and in real time due to GPS application. PMU installed at a bus bar measures not only current and voltage phasors at the respective bus bar but ensures the values of voltages at the incident bus bars that are obtained due to passive line parameters and values of currents in the observed lines. The measurements provided by the PMU installed in the dispersed locations due to their time stamps are widely used in post-disturbance analysis, advanced relay protection, stability determination, false data identification, online control and monitoring. As a rule PMU is incorporated with digital devices of relay protection, electric meters and fault recorders. However the architecture applied to obtain maximal effect from synchronized measurement technology beside the PMU must contain communications networks and Phasor Data Concentrator (PDC), that results in the considerable increasing of investment in such technologies. Further in this article the complex of PMUs, network communications and PDC will be called by the general concept PMU.

In this context the main purpose of the methods for PMU placement is to identify minimal number of PMUs to be installed and points of PMU's optimal placement. This objective should be achieved in order to ensure the complete observability, data redundancy and costs minimizing.

The complete observability is achieved when all the bus bars in power system are observed at least by one PMU.

Data redundancy is used as a feature that describes the number of individual PMUs that monitors the relative bus bar (node). It is considered that at the bus bar with zero injection (where the sum of current phasors is equal to zero) it is not necessary to install PMU for voltage phasor determination.

Most theoretical implementations of PMU optimal placement are focused only on minimizing the number of PMU in normal operating regime of power system, but diverse contingencies are not taken into considerations. However PMU placement has to ensure functional security in case of eventual outage of PMU or communication network, or in case of limited number of channels for measurements. „Phasing or multistage” also represents an important criterion for optimal PMU placement, because sometimes PMU installation is carried out on the different time stages.

The main objective of this paper is to analyze the methods of optimal PMU placement. In order to analyze the opportunities of the proposed methods, there were compared the results of their application for test system IEEE-14 with 14 bus bars. The results are compared by different criteria to determine if the methods are compatible in different conditions.

## II. THE CLASSIFICATION OF THE PMU OPTIMAL PLACEMENT METHODS

According to [3] all methods of optimization can be classified in 3 groups: mathematical methods (based on deterministic techniques), heuristic methods and meta-heuristic methods.

Heuristic methods often called approximation algorithms are used to accelerate the process of finding a reasonable solution under conditions when other methods are impractical. The optimal result can not be guaranteed by using these methods.

1. *Depth-First Search Algorithm*
2. *Minimum Spanning Tree*
3. *Greedy algorithm*

Meta-heuristic methods, which are an improvement on the heuristic methods, involve intelligent search processes that can deal with discrete variables and non-continuous cost functions. Basically, this method combines randomized algorithm and local optimization algorithm to solve the optimal problem [3].

### 1. Genetic Algorithm

### 2. Simulated Annealing

### 3. Particle Swarm Optimization

Mathematical methods widely use integer programming and numerical algorithms of calculation. Unknown variable is introduced in linear equation that can be solved with or without numerous iterations.

#### 1. Integer Linear Programming

#### 2. Method of determinant measures

Besides these methods there are widely used hybrid methods that are based on several techniques or for instance the algorithm of „fuzzy logic”.

Due to the structure of power systems a lot of optimal placement methods implement the graph theory, for example Depth-First Search (DFS) algorithm. The strategy of searching in depth involves the traversing an edge from the present node to the undiscovered node. When all edges are explored it returns to the node that has led to the exploration of the current node. The process is repeated until all the nodes are explored and the complete observability is achieved.

According to this method the first PMU has to be placed at the bus with the largest number of connected branches and if there was more than one bus with this characteristic, one is randomly chosen.

*Minimum Spanning tree method* is the modification of the depth-first search algorithm and it assumes a number of passages along the branches of the minimal spanning tree for the purpose of installing PMU in the nodes on this route PMU to provide complete observability. For example, for test system IEEE-14 the search procedure starts at the root node 1, so the first PMU should be installed at node 2. In [5] is mentioned that that the PMUs are physically separated 3 buses from each other along the chosen path. Thus the next PMU should be installed at node 9, then at node 6. So all nodes and links are observable except node 8, which becomes observable when the last PMU is installed at node 7 (see fig. 1).

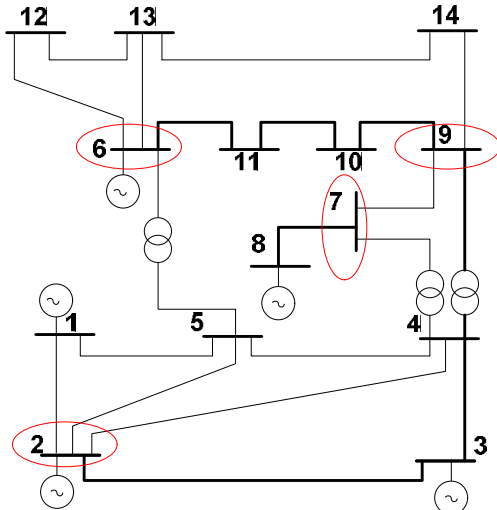


Fig. 1. Illustration of the minimal spanning tree method with IEEE-14

Another method that analyzes power system topology is based on *greedy algorithms*, where the local optimal solution is considered to be the global optimal solution. Although this method effectively reduces the computational efforts and time for calculation, the solution found by one of the algorithms "greedy" can not be called optimal. Figure 2 illustrates the application of the greedy algorithm called minimal graph coloring, which provide the approximate but not optimal solution in comparison with other heuristic methods.

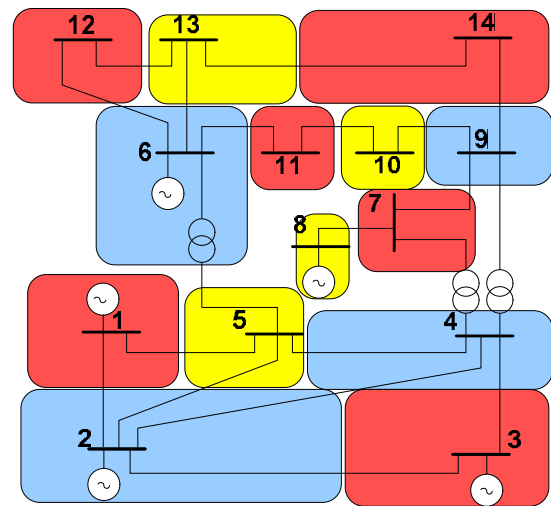


Fig. 2. Illustration of the minimal graph coloring method with IEEE-14

More advanced solutions for the problems of optimization are ensured by meta-heuristic methods that very often use unexpected concepts, inspired by natural evolution. For example *genetic algorithm* performs specific operations in a reproduction process governed by genetic operators. The new solutions are created by selection and recombination of existing chromosomes in order to optimize a specific evaluation functions to each individual problem. There are used some specific procedures in the process of finding the optimal solution:

- the selection is a genetic operator that sets strings of current population to be chosen to transmit their genes to the next generation;
- crossover is an operator necessary for the construction of new individuals in population;
- mutation allows to find new solutions in the algorithm and protect it from loss of information as a result of inappropriate crossovers.

The structure of the genetic algorithm is given below:

1. Initialize the population of chromosomes;
2. Evaluate each chromosome of the population. Select parents for new populations;
3. Create a new generation of chromosomes by crossover the selected chromosomes and using genetic operators;
4. Delete the original members of the population, by replacing with new generation;
5. Ne chromosomes are evaluated and inserted into the new population;

6. If time search is not finished, go to step 3. Otherwise, the algorithm stops execution.

This algorithm introduces several additional parameters (maximum number of incident branches, number of current phasors that must be measured by the installed PMU). References [3], [7], [9] and [1] confirm the efficiency and accuracy of this method, although it is very sophisticated in comparison with other methods and in plus it can provide non-optimal solution or in contradictory nodes.

Other method based on the bioinformation technologies is called particle swarm optimization. It is the optimization method that ensures the procedure of finding the solution based on population in which particles changes their positions in time. Particles moves in multidimensional space of solutions. This method ensures the reduction of time for solving, but at the same time it requires to search in advance the initial locations for PMU, node with only one connected branch and nodes with zero injection. Also during the calculation it is necessary to control estimation function and particles' velocity.

The technical overview of the meta-heuristic methods shows that besides the techniques above analyzed there are other methods based on nature evolution and bioinformation specified with the same advantages, disadvantages and stages of solving (simulated annealing, Tabu method, differential evolution method, immune algorithm, ant colony optimization, bacterial algorithm).

The group of the deterministic methods are widely spread and developed at the advanced level. The problem of optimal PMU placement can be solved with the help of integer leinear programming if there is determined the function of optimization:

$$\text{Min} \sum_{j=1}^n w_j \cdot x_j, \quad (1)$$

with conditions:

$$\begin{aligned} C_{PMU} \cdot X &\geq b_{PMU} \\ \mathbf{X} &= [x_1, x_1, x_2 \dots x_n]^T, \quad x_i \in \{0,1\} \\ b_{PMU} &= [1, 1 \dots 1]_{n \times 1}^T \end{aligned} \quad (2)$$

Here  $n$  represents the total number of the nodes,  $w_j$  - PMU installation cost at the node  $j$ ,  $X$  - binary variable vector, that gets value 1 when in the respective node is required PMU and 0 - otherwise. Vector  $b_{PMU}$  specifies the criterion of the observability in each node. The binary matrix of the connectivity  $C_{PMU}$  can be obtained from the matrix of admittances by transforming its elements in binary form.

For test system IEEE-14 matrix  $C_{PMU}$  has following form:

$$C_{PMU} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (3)$$

Thus it is necessary to solve the system of equations (4).

$$f(X) = \begin{cases} x_1 + x_2 + x_5 &\geq 1 \\ x_1 + x_2 + x_3 + x_5 &\geq 1 \\ x_2 + x_3 + x_4 &\geq 1 \\ x_3 + x_4 + x_5 + x_7 + x_9 &\geq 1 \\ x_1 + x_2 + x_4 + x_5 + x_6 &\geq 1 \\ x_5 + x_6 + x_{11} + x_{12} &\geq 1 \\ x_4 + x_7 + x_8 + x_9 &\geq 1 \\ x_7 + x_8 &\geq 1 \\ x_4 + x_7 + x_9 + x_{10} + x_{14} &\geq 1 \\ x_9 + x_{10} + x_{11} &\geq 1 \\ x_6 + x_{10} + x_{11} &\geq 1 \\ x_6 + x_{12} + x_{13} &\geq 1 \\ x_{12} + x_{13} + x_{14} &\geq 1 \\ x_9 + x_{13} + x_{14} &\geq 1 \end{cases} \quad (4)$$

There are different tools for solving system (4) and in this article they are not discussed. The method of linear programming is very simple even in case of analyzing different scenarios of power grid development, that is why it will be used further for contingencies analyze (presence of the nodes with zero injection, outage in the measurement system).

In order to simplify and accelerate calculations based on mathematical model can be applied the *method of determinant measures*. The final purpose of optimal PMU placement is to determine regime parameters for power system, thus the structure of PMU installation in grid nodes will be reflected in admittance matrix  $Y$  from the system of nodal equations. In [4] it is shown that if matrix  $Y$  has special form, then the calculations will be more simplified.

Thus it is rational to transform matrix  $Y$  in order to allocate the submatrix with triangular form. Decomposition of the matrix  $Y$  provides determinant measures. They represent the minimal number of searched measures (here nodes with PMU), that allow to determine other unknown values (voltages in other nodes and currents in branches) by solving equations with one single unknown value. If there are known determinant measures, determinable values can be found by any solving method. Actually there are elaborated different algorithms of the selection the determinant measures [4], [6] and [8], for instance by equivalent transforming, that involves consequent transpositions of rows and columns and simultaneous permutations of the elements in submatrixes of determinant and determinable measures. Other algorithm of selection the determinant measures is illustrated for test system IEEE-14 and it involves line profile exclusion from the graph of the grid. Thus the nodes of the graph are divided in two groups –  $F$  and  $L$ . The group  $F$  contains nodes which participate in allocation in the initial graph trees with the simple construction, the rest of the nodes are included in group  $L$ . Group  $L$  in its turn is divided in 2 subsets  $N$  and  $R$ , where  $N$  contains nodes that are connected to the tree by only one branch and the rest of the nodes are included in subset  $R$ .

The main advantage of application the method of determinant measures is the simplified calculation effectuated without iterations, by solving equations with one single unknown value called the determinable measure. In this context besides the date provided by PMU installed in nodes 2, 6, 7 and 9 is required to use conventional measurements (ensured by telemecanics or SCADA). So for the test system IEEE-14 it is involved application of the conventional measurements in nodes 1, 10 and 12 (see fig. 3).

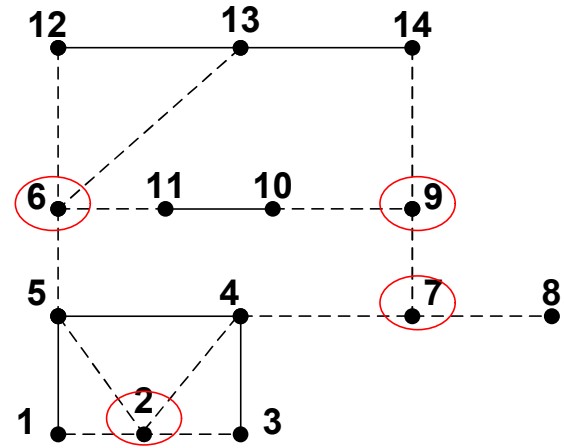
The same algorithm applied for test system IEEE-30 shows that introduction in calculations the conventional measurements from the 7 nodes reduces the number of installed PMU from 10 (according to [2]) to 6, thus the PMU should be installed in nodes with maximal number of connected branches, i.e. nodes 4, 6, 10, 12, 23 and 27.

Taking into account these considerations it is rational to develop this method: in order to reduce calculation time and to ensure optimal solution is required to elaborate restrictions and evaluation function.

When analyzing methods and algorithms above mentioned can be made a conclusion that all the tools of optimal PMU placement can be classified as:

- methods for investigating topological properties – heuristic methods;
- random selection methods – meta-heuristic methods;
- numerical methods – mathematical methods.

The heuristic and meta-heuristic methods very often provide acceptable results in stead of optimal results, that require to improve restrictions and evaluation function, in its turn this may bring to sophistication of the solving techniques. From all mentioned methods numerical methods ensure the most reliable results. In plus these techniques can be applied for analyzing the dependence of the number of PMU from the presence of contingencies.



Nr. node	Set F								Set L							
	2	6	7	9	1	10	12	3	4	5	11	13	14	8		
2	1	0	0	0	1	0	0	1	1	1	0	0	0	0		
6	0	1	0	0	0	0	1	0	0	1	1	1	0	0		
7	0	0	1	1	0	0	0	0	1	0	0	0	0	1		
9	0	0	1	1	0	1	0	0	1	0	0	0	1	0		
1	1	0	0	0	1	0	0	0	0	1	0	0	0	0		
10	0	0	0	1	0	1	0	0	0	0	1	0	0	0		
12	0	1	0	0	0	0	1	0	0	0	0	1	0	0		
3	1	0	0	0	0	0	0	1	1	0	0	0	0	0		
4	1	0	1	1	0	0	0	1	1	1	0	0	0	0		
5	1	1	0	0	1	0	0	0	1	1	0	0	0	0		
11	0	1	0	0	0	1	0	0	0	0	1	0	0	0		
13	0	1	0	0	0	0	1	0	0	0	0	1	1	0		
14	0	0	0	1	0	0	0	0	0	0	0	1	1	0		
8	0	0	1	0	0	0	0	0	0	0	0	0	0	1		

Fig. 3. Illustration of the determinant measures method with IEEE-14

### III. CONTINGENCIES IN THE PROBLEM OF OPTIMAL PMU PLACEMENT

In the formulation of the problem can be introduced one of the condition specific to the real state of the power system:

- The presence of the bus bars with zero injection;
- PMU outage resulted in loss of the transmission channel;
- Electric line outage;
- PMU or electric line outage.

The influence of these contingencies will be illustrated with the help of integer linear programming method.

If in the model of optimal placement are introduce bus bars with zero injection then the total number of PMU necessary for complete observability will be reduced considerably. It is possible due to the properties of the bus

bars with zero injection: 1) when buses, which are incident to an observable zero injection bus, are all observable except one, the unobservable bus will also be identified as observable by applying the Kirchhoff law at zero injection bus; 2) when buses incident to an unobservable zero injection bus are all observable, the zero injection bus will also be identified as observable by applying the Kirchhoff law at zero injection bus. These considerations result in introduction into formulas (2) and (4) the parameter of zero-injection as an additional value. For test system IEEE-14 which contains only one zero injection bus (node 7) the restrictions will be determined as:

$$f(X) = \begin{cases} x_1 + x_2 + x_5 \geq 1 \\ x_1 + x_2 + x_3 + x_5 \geq 1 \\ x_2 + x_3 + x_4 \geq 1 \\ x_3 + x_4 + x_5 + x_7 + x_9 \geq u_4 \\ x_1 + x_2 + x_4 + x_5 + x_6 \geq 1 \\ x_5 + x_6 + x_{11} + x_{12} \geq 1 \\ x_4 + x_7 + x_8 + x_9 \geq u_7 \\ x_7 + x_8 \geq u_8 \\ x_4 + x_7 + x_9 + x_{10} + x_{14} \geq u_9 \\ x_9 + x_{10} + x_{11} \geq 1 \\ x_6 + x_{10} + x_{11} \geq 1 \\ x_6 + x_{12} + x_{13} \geq 1 \\ x_{12} + x_{13} + x_{14} \geq 1 \\ x_9 + x_{13} + x_{14} \geq 1 \end{cases} \quad (5)$$

$$u_4 + u_7 + u_8 + u_9 \geq 3 \quad (6)$$

Taking into consideration these changes the will be obtained the reduced number of nodes for PMU installation (for IEEE-14 PMU should be installed in nodes 2, 6 and 9).

The reliability of the monitoring system is ensured when every bar in the power system is observed at least by one PMU. If in the model of optimization it is taken into consideration the possibility of PMU outage, this results in multiplying the right part of the expressions (4), for example for node 1 is obtained:

$$x_1 + x_2 + x_5 \geq 2 \quad (7)$$

In order to ensure complete observability in the post-disturbance regime, when electric line is defected and the path of observing is modified, it is required to modify also the respective elements in the connectivity matrix  $C_{PMU}$  for new structure of the power grid.

If the line outage results in loss of measurements from PMU it is necessary to take into consideration both changes in matrix  $C_{PMU}$  and multiplying in observability criterion  $b_{PMU}$ .

#### IV. SIMULATION RESULTS

The above mentioned considerations were tested on test systems IEEE-14 and IEEE-30 for 5 different scenarios referred to the optimal location of PMU:

1. *in normal conditions (presence of the measurements from PMU and conventional measurements);*
2. *with presence of zero injection bus bars;*
3. *PMU outage;*
4. *electric line outage;*
5. *PMU or electric line outage.*

TABLE I. CALCULATION RESULTS OF OPTIMAL PMU PLACEMENT

Test system	Scenario	Number of PMU for complete observability	PMU location
IEEE-14	Nr. 1	4	2, 6, 7, 9
	Nr. 2	3	2, 6, 9
	Nr. 3	7	2, 4, 5, 6, 9, 10, 13
	Nr. 4	7	1, 3, 6, 8, 9, 11, 13
	Nr. 5	8	1, 2, 4, 6, 8, 9, 10, 13
IEEE-30	Nr. 1	10	2, 4, 6, 9, 10, 12, 15, 18, 25, 27
	Nr. 2	7	3, 5, 10, 12, 18, 23, 28
	Nr. 3	15	1, 3, 5, 7, 10, 12, 13, 15, 16, 19, 20, 24, 25, 27, 29
	Nr. 4	13	1, 3, 5, 10, 11, 13, 14, 15, 16, 19, 23, 26, 30
	Nr. 5	17	1, 3, 5, 7, 10, 11, 12, 13, 15, 16, 19, 20, 23, 24, 26, 27, 30

In the scenarios 3, 4 and 5 the presence of zero injection buses is taken into consideration. The calculation results are shown in table I.

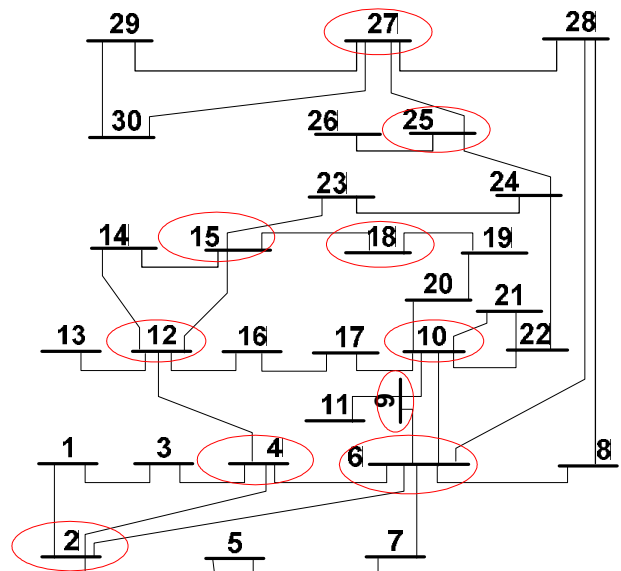


Fig. 4. Illustration of the PMU placement with IEEE-30 (scenario nr.1)

## V. CONCLUSION

In this paper there were examined different methods of optimal placement of the synchronized measurement technologies. It is worth noting that the results obtained by different methods practically coincide. Despite this fact the application of the heuristic and meta-heuristic methods is reasonable in case when there are preset desirable locations for PMU placement or when it is required power system division into island independent but observable. The deterministic methods (integer linear programming and method of determinant measures) are characterized with universality in application, however the method of determinant measures might be performed under contingency constrains in order to develop the best variant of evaluation function and restrictions.

In plus there was studied the influence of contingencies that can appear in real power system upon the optimal number of PMU and their placement. The obtained results show that neither reliability nor security of power grid can be ensured if contingencies mentioned above are neglected. These additional factors can reduce the optimal number of PMU on the one hand (as zero injection buses do) or can raise their number and even location (like outages of the PMU or/and electric line do).

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