THE GENERAL SOLUTION OF PROBLEM ON INTERACTION BETWEEN WAVE AND STREAM IN THE UNDULAR FRAMES

B. Mironov, PhD, assoc.prof.
Technical University of Moldova

INTRODUCTION

The undular reference frames represent interest at examination of waves behavior in absence of heterogeneous objects in medium. In this case as tools serves other waves in same medium. Thus, we term undular frame, the system, in which as time etalon serve the period of standing or semi-standing wave in a fixed point, and length etalon is equal to distance between two points with an identical phase. In paper [1] we have shown, that the transition from one such system in another is implemented in correspondence with Lorentz transformations.

In the previous article [2] the particular solution of a problem was given for the force which act on the wave on stream border, when the wave goes along border. In present paper we shall give the general solution of this problem for a case, when the wave goes under an arbitrary corner in relation to stream border. Finally the obtained solution is extended on the case, when the velocity of medium varies smoothly depending on coordinate.

2. THE PROBLEM DEFINITION

Input data:

- the study area - continuous medium, in which can exist streams or differences of velocity. We shall term this medium as the continuum;
- the object of examination - the differences of velocity in continuum;
- the tool used for examinations - the wave-tool representing domain with redundant pressure (density) in comparison with pressure in a unperturbed continuum.

Problem: to determine, which parameters characterise the interactions of the wave-tool with streams, and how these parameters are interlinked among themselves.

3. INTERACTION OF THE WAVE-TOOL WITH DIFFERENCE OF VELOCITY

In the article [2] we have shown, that the wave can detect a pressure drop. In the field of a pressure drop, on a wave act the force

$$F_T = \pm S \frac{\rho_T}{\rho_0} \Delta p . \qquad (1)$$

Here S - is sectional area of wave-tool in the domain of pressure drop, Δp - pressure drop, ρ_T - redundant density caused by the wave - tool, ρ_0 - density of medium. Unfortunately, in the quoted article the normalization factor ρ/ρ_0 was missed. The sign in expression (1) is determined by the sign of redundant density.

As was shown in article [1], the wave - tool "can not detect" the motion velocity concerning the medium, in which it exist. But we will show, that the wave-tool is capable "to detect" the drop of velocity in medium and we shall calculate force, which act to the wave-tool on the stream border.

Let suppose that, the wave-tool, which carries surplus of a continuum, goes along the axis x and intersect the flux border (figure 1). We must choose three reference frames:

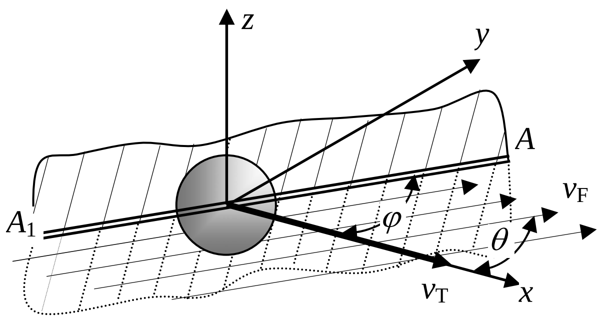

Figure 1. The wave intersect the flux border, all parameters are measured in a laboratory frame.

- the laboratory system, for definiteness we shall consider, that concerning it, the continuum is immobile;
- reference frame bound with a flux;
- reference frame of the wave-tool.

In laboratory system are used the following notations: v_F - flux velocity; v_T - wave-tool velocity; φ - angle between flux border and direction of wave-tool motion; θ - angle between flux velocity v_F and axis x in laboratory system. The angles φ and θ are equal in laboratory system among themselves, but in other systems they can be not equal among themselves. The same values measured in the flux system, we shall mark by one accent. If these values are measured in system of the wave-tool, they will be marked by two accents.

For excluding the influence of a static pressure drop, which was studied in the previous section, we consider, that in laboratory system, the pressure of a continuum p_C is equal to flux pressure p_F i.e. $p_C=p_F=p_0$. Thus, if the wave-tool rest in a laboratory frame, the resulting force acting on it on flux border, is equal to zero.

System, in which the viewed segment of length has maximum length, we shall term as own system. Let's consider, that in laboratory system the pressure is isotropic.

The transitions between reference frames are accomplish with help of coefficients.

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\left(v/c\right)^2}}, \qquad (2)$$

If the transition is accomplish from own system, the length of a segment oriented along axis x is necessary to multiply onγ. At inverse transition the segment length is divided on γ.

Now we can formulate a problem as follows: **we must to determine value and direction of force acting on the wave-tool on flux border in laboratory system, as function from flux velocity v_F and wave-tool velocity v_T.**

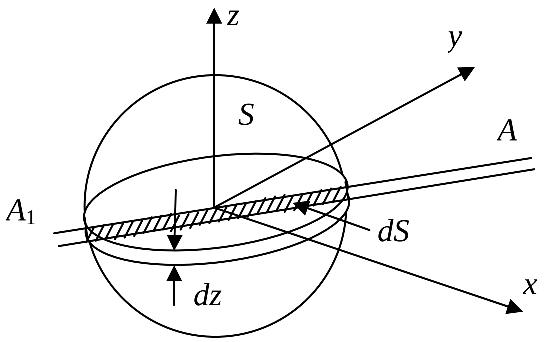

Figure 2. The layer of the wave-tool having thickness *dz*, for which we determine the force acting on flux border

At first we shall determine the force which act on the layer, having the thickness *dz*, and clipped by planes, parallel to plane *xy* (figure 2.). As the problem is symmetric concerning the plane *xy*, it is enough to determine the force *F*, which act on the layer in this plane.

For the solution of posed problem the following algorithm is applicable:

1. we shall calculate force, which act on the wave-tool from the "stationary" continuum in wave-tool system;
2. we shall calculate force, which act on the wave-tool from the flux in wave-tool system;
3. we shall determine the resulting force in the wave-tool frame;
4. we shall convert the resulting force in laboratory system.

The force, which act on the wave-tool from the continuum, is equal to integral from pressure on the surface of the upper arc MM_1 multiplied on dz (figure 3). By virtue of symmetry, the tangential component is compensated. Therefore force acting on the wave-tool from the continuum in laboratory system is equal

$$F_C = 2\frac{\rho}{\rho_0} p_0 R dz = \frac{\rho}{\rho_0} p_0 dS .$$

Here $dS = Rdz$. The projections of this force on an axis x and y will be:

$$F_{Cx} = \frac{\rho}{\rho_0} p_0 dS \sin\varphi \quad \text{and} \quad F_{Cy} = \frac{\rho}{\rho_0} p_0 dS \cos\varphi .$$

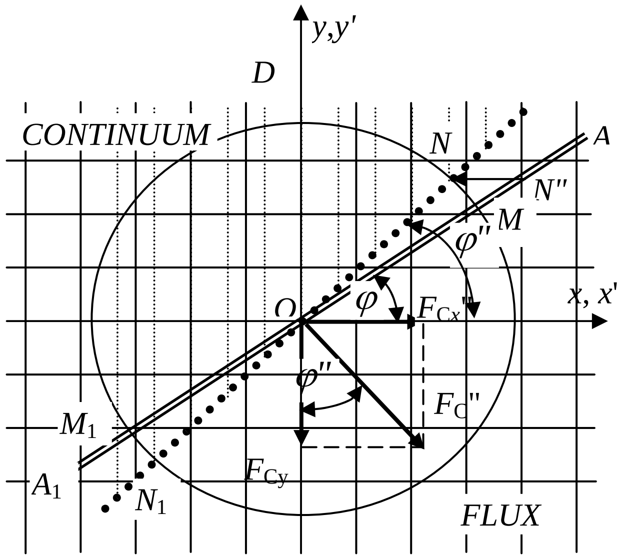

Figure 3. The changing of position of flux border, when transition from laboratory system in a wave-tool frame of reference.

At transition in wave-tool frame, the coordinate scale tests relativistic compression in the axis x direction, and in the direction of an axis y gauge remains constant. Thus the point N'' is displaced in the position of N, and the angle between the axis x and flux border φ is transformed in φ''. Coefficient of compression along an axis x is equal to

$$\gamma_T = \frac{1}{\sqrt{1-\beta_T^2}}, \qquad (3)$$

where $\beta_F = \dfrac{v_F}{c}$.

Hence, in system of the wave-tool the component forces acting from "stationary" continuum will be:

$$F_{Cx}" = \frac{\rho}{\rho_0} p_0 dS \sin\varphi = 2\frac{\rho}{\rho_0} dz(p_0)(R\sin\varphi), \quad (4)$$

$$F_{Cy}" = -\frac{\rho}{\rho_0} p_0 dS\gamma_T \cos\varphi = = -2\frac{\rho}{\rho_0} dz(\gamma_T p_0)(R\cos\varphi) \quad (5)$$

As was already marked, at transition from laboratory system in wave-tool frame, the point *N* " is displaced in the position *N*. That is, as it is visible from figure 3, the length of border segment, with which the wave-tool interact, varies. For brevity the length of border segment, with which the wave-tool interact, we shall name as effective length. Thus, the transition between frames is linked to the change of three values: continuum pressures, angle between border and axis *x* , and effective length of border.

According to item 3 of our algorithm, we must summarise two forces acting on the wave-tool. Namely: force acting from the continuum and force, acting from the flux. In this case, in the wave-tool frame, the effective length of border, is common for both "fixed continuum " and flux, while the angles and pressure will differ. For this reason in expressions (4) and (5) we should distinct three marked components. Therefore we express formulas (4) and (5) in the terms of the wave-tool, that is, we shall express through values with two accents.

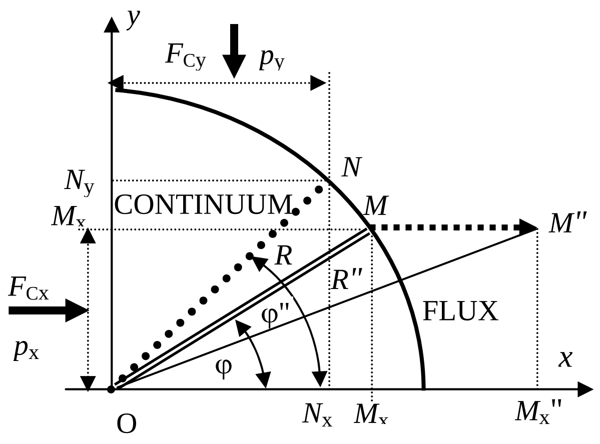

Figure 4. To determination of separate transformation of effective border length and pressure

The flux border is located in the first and third quadrant symmetrically relatively of frames origin *O*. Therefore it is enough to carry out the analysis for the first quadrant (figure 4). The transition from the laboratory frame into wave-tool frame can be described in two stages: the squeezing of border segment, with which interreacts the wave-tool and its rotation.

If in laboratory system the effective length is equal to *R* = *OM*, then, in wave-tool frame, its projection to an axis *x* will be longer in γ_T times. At such transformation the point *M* transfers in the point *M "* and, hence, *R* is conversed in *R"*. So, the effective value of radius in wave-tool frame:

$$R" = R\sqrt{\cos^2\varphi + \gamma_T^2 \sin^2\varphi} = KR\,. \quad (6)$$

Here we have designated $K = \sqrt{\cos^2\varphi + \gamma_T^2 \sin^2\varphi}$. Simultaneously border turns, so, that the angle φ is transferred in φ". So

$$\mathrm{tg}\varphi" = \gamma_T \mathrm{tg}\varphi\,. \quad (7)$$

By using formula (7) in known relations:

$$\sin\varphi" = \frac{\mathrm{tg}\varphi"}{\sqrt{1+\mathrm{tg}^2\varphi"}} \quad \text{and} \quad \cos\varphi" = \frac{1}{\sqrt{1+\mathrm{tg}^2\varphi"}} \quad (8)$$

we receive

$$\sin\varphi" = \frac{\gamma_T \mathrm{tg}\varphi}{\sqrt{1+\gamma_T{}^2\mathrm{tg}^2\varphi}} = \frac{\gamma_T \sin\varphi}{\sqrt{\cos^2\varphi + \gamma_T{}^2\sin^2\varphi}} \quad (9)$$

$$\cos\varphi" = \frac{1}{\sqrt{1+\gamma_T{}^2\mathrm{tg}^2\varphi}} = \frac{\cos\varphi}{\sqrt{\cos^2\varphi + \gamma_T{}^2\sin^2\varphi}} \,. \quad (10)$$

Taking into account expressions (6), (9) and (10) it is possible to write down the formulas for transformations of projections *R* on an axis *y* and *x* accordingly:

$$R\sin\varphi = R"\frac{\sin\varphi"}{\gamma_T}, \quad (11)$$

$$R\cos\varphi = R"\cos\varphi"\,. \quad (12)$$

Let's set (11) and (12) in (4) and (5), we shall receive the projections of force acting from the "stationary continuum" in the wave-tool frame:

$$F_{Cx}" = F_{Cx} = \frac{\rho}{\rho_0} p_0 dS \sin\varphi = = 2\frac{\rho}{\rho_0} dz(p_0)\left(\frac{\sin\varphi"}{\gamma_T}\right)R" \quad (13)$$

$$F_{Cy}" = -\frac{\rho}{\rho_0} p_0 dS\gamma_T \cos\varphi = = -2\frac{\rho}{\rho_0} dz(\gamma_T p_0)(\cos\varphi")R" \,. \quad (14)$$

At description of force components acting from flux, the expressions which feature transformations of pressure and angle will vary, that is expression in brackets. Effective radius *R*" will remain constant.

Thus, we can use the similar formulas at definition of force acting from the flux.

For determination of force acting to the wave-tool from the flux in the wave-tool frame is necessary to re-count the flux pressure from laboratory system in the wave-tool frame. Both these systems are not own in relation to the flux. The problem is, that through coefficients γ (2) it is possible to realize a transition between two systems, one of which should be own. For this reason at first we should determine flux density in its own reference frame, and then calculate flux density in wave-tool frame.

From the equilibrium condition on the border in the laboratory frame, follows, that: $p_C=p_F=p_0$. From here pressure of the flux in its own system:

$$p_F' = \frac{p_F}{\gamma_F} = \frac{p_0}{\gamma_F}$$

Where $$\gamma_F = \frac{1}{\sqrt{1-\beta_F^2}} \qquad (15)$$

and $\beta_F = \frac{v_F}{c}$.

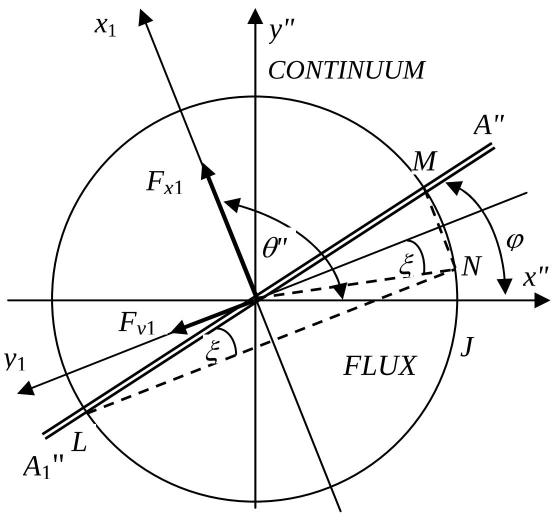

Figure 5. To definition of force acting on the wave-tool from the flux

We are able to determine pressures (and consequently also forces), which act on the wave-tool along the direction of its motion and in the perpendicular direction. But in this case the direction of flux motion in the wave-tool frame does not coincide with axes x and x''. The flux goes under the angle θ' to an axis x'' in the wave-tool frame. Therefore, for finding force, which act from the flux, we use an auxiliary frame x_1y_1. This system is linked to the wave-tool, and is oriented in such manner, that the axis x_1 coincides with the direction of the flux motion relatively of wave-tool, i.e. with v_F''. The plane x_1y_1 coincides with the planes $x''y''$ and xy. As we can see on figure 5, the axis x_1 is turned relatively x'' and x on the angle θ'.

Thus, from the "point of view" of the wave-tool the "stationary continuum" goes along the axis x'', and the flux goes along the axis x_1. Coefficient of gauge reduction along the axis x_1 when transit from frame of flux to wave-tool frame:

$$\gamma_F'' = \frac{1}{\sqrt{1-(\beta_F'')^2}}. \qquad (16)$$

Here $\beta_F'' = \frac{v_F''}{c}$ and v_F'' - velocity of the flux motion relatively wave-tool frame. Let's designate v_L- velocity of the motion of laboratory system relatively the wave-tool. We will found the velocity of the flux motion relatively of wave-tool frame according to the relativistic law of the velocity addition and taking into account, that $v_L = -v_T$, we can note:

$$\beta_F'' = \frac{\sqrt{\beta_F^2+\beta_T^2-2\beta_F\beta_T\cos\theta-\beta_F^2\beta_T^2\sin^2\theta}}{1-\beta_F\beta_T\cos\theta}. \qquad (17)$$

Having substituted (17) in (16), after simple transformations we shall receive:

$$\gamma_F'' = \frac{1-\beta_F\beta_T\cos\theta}{\sqrt{(1-\beta_F^2)(1-\beta_T^2)}}$$

Or in view of expressions (3) and (15):

$$\gamma_F'' = \gamma_F\gamma_T(1-\beta_F\beta_T\cos\theta). \qquad (18)$$

Thus, for transversal and longitudinal component pressure acting on the wave-tool from the flux, it is possible to note:

$$p_{y1} = p_F'\gamma_F'' = \frac{p_F}{\gamma_F}\gamma_F'' = p_0\frac{\gamma_F''}{\gamma_F}$$

and

$$p_{x1} = p_{Fx}' = \frac{p_{Fx}}{\gamma_F} = \frac{p_0}{\gamma_F}.$$

Then by analogy to expressions (13) and (14) and taking into account (6), the components of force acting from the flux on the considered layer of the wave-tool, will be:

$$F_{y1} = 2\frac{\rho_T}{\rho_0}dz\left(\frac{p_0}{\gamma_F}\gamma_F''\right)(\sin\xi)KR,$$

$$F_{x1} = 2\frac{\rho_T}{\rho_0}dz\left(\frac{p_0}{\gamma_F}\right)\left(\frac{\cos\xi}{\gamma_F''}\right)KR$$

As well as in expressions (13) and (14) in brackets we have the parameters, linked with transformations of pressure in the flux and slope angle of border in relation to the axis y_1 of auxiliary frame. In this case is more convenient to counting the angle ξ from the axis y_1.

In view of expression (18), we shall receive:

$$F_{y1} = K\frac{\rho_T}{\rho_0} dSp_0\gamma_T\left(1-\beta_F\beta_T\cos\theta\right)\sin\xi \,, \quad (19)$$

$$F_{x1} = K\frac{\rho_T}{\rho_0} p_0 \frac{\cos\xi}{\gamma_F^2\gamma_T\left(1-\beta_F\beta_T\cos\theta\right)} dS \,, \quad (20)$$

where $dS = 2dzR$ - area of the border, cut by the viewed layer of the wave-tool (figure 5).

Let's express functions of the angle ξ through functions θ' and φ''. As it is visible from figure 5,

$$\xi = \varphi'' - \left(\theta'' - \frac{\pi}{2}\right) = \varphi'' - \theta'' + \frac{\pi}{2}\,.$$

From here: $\sin\xi = \dfrac{1+\mathrm{tg}\varphi''\ \mathrm{tg}\theta''}{\sqrt{\left(1+\mathrm{tg}^2\varphi''\right)\left(1+\mathrm{tg}^2\theta''\right)}}$

And $\cos\xi = -\dfrac{\mathrm{tg}\varphi'' - \mathrm{tg}\theta''}{\sqrt{\left(1+\mathrm{tg}^2\varphi''\right)\left(1+\mathrm{tg}^2\theta''\right)}}\,. \quad (21)$

For determination of the angle θ'' we apply the formula for angle transformation. Let's v_F is flux velocity in laboratory system and v_L is velocity of laboratory system relatively wave-tool. At the relativistic addition, the angle between flux velocity in wave-tool frame v_F'' and axis x will be:

$$tg\theta'' = \frac{\beta_F\sqrt{(1-\beta_T^2)}\sin\theta}{\beta_F\cos\theta - \beta_T}\,,$$

or, taking into account (3):

$$tg\theta'' = \frac{\beta_F\sin\theta}{\gamma_T(\beta_F\cos\theta - \beta_T)}\,. \quad (22)$$

Let's substitute in (21) expressions: (7) and (22) also we shall take into account, that: $\theta = \varphi$.

We receive:

$$\sin\xi = U\frac{\gamma_T\left(\beta_F - \beta_T\cos\theta\right)}{\sqrt{\left[\gamma_T^{\ 2}(\beta_F\cos\theta - \beta_T)^2 + \beta_F^{\ 2}\sin^2\theta\right]}}$$

$$\cos\xi = U\frac{\gamma_T^{\ 2}\beta_T\sin\theta(1-\beta_T\beta_F\cos\theta)}{\sqrt{\left[\gamma_T^{\ 2}(\beta_F\cos\theta - \beta_T)^2 + \beta_F^{\ 2}\sin^2\theta\right]}}\,.$$

We have designated here

$$U = \frac{1}{\sqrt{\left(\cos^2\theta + \gamma_T^{\ 2}\sin^2\theta\right)}}\,.$$

In view of expression (3) and after the series of transformations we shall have:

$$\sin\xi = \\ = \frac{U\left(\beta_F - \beta_T\cos\theta\right)}{\sqrt{\left(\beta_F^{\ 2} + \beta_T^{\ 2} - 2\beta_F\beta_T\cos\theta - \beta_F^{\ 2}\beta_T^2\sin^2\theta\right)}}\,, \quad (23)$$

$$\cos\xi = \\ = \frac{U\gamma_T\beta_T\sin\theta(1-\beta_T\beta_F\cos\theta)}{\sqrt{\left(\beta_F^{\ 2} + \beta_T^{\ 2} - 2\beta_F\beta_T\cos\theta - \beta_F^{\ 2}\beta_T^2\sin^2\theta\right)}}\,. \quad (24)$$

By substituting (23) and (24) in the formulas (19) and (20), we shall receive:

$$F_{y1} = \frac{\rho_T}{\rho_0}\times \\ \times\frac{dSp_0\gamma_T\left(1-\beta_F\beta_T\cos\theta\right)\left(\beta_F - \beta_T\cos\theta\right)}{\sqrt{\beta_F^{\ 2} + \beta_T^{\ 2} - 2\beta_F\beta_T\cos\theta - \beta_F^{\ 2}\beta_T^2\sin^2\theta}}\,, \quad (25)$$

$$F_{x1} = \frac{\rho_T}{\rho_0} p_0 \frac{1}{\gamma_F^2}\times \\ \times\frac{dS\beta_T\sin\theta}{\sqrt{\left(\beta_F^{\ 2} + \beta_T^{\ 2} - 2\beta_F\beta_T\cos\theta - \beta_F^{\ 2}\beta_T^2\sin^2\theta\right)}} \quad (26)$$

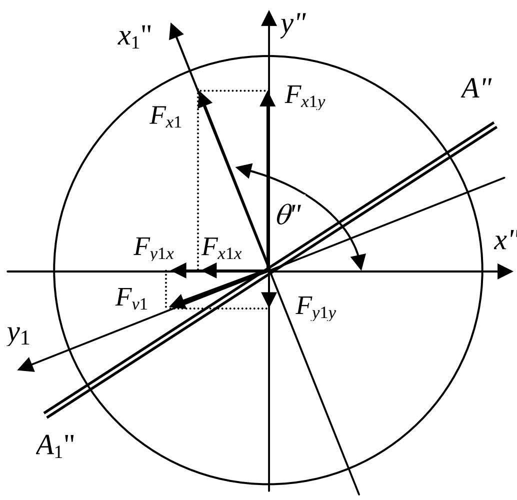

Figure 6. The projections of force acting on the wave-tool from the flux.

The formulas (25) and (26) describe projections of force F_F on an axis y_1 and x_1 of auxiliary frame. Let's remind, F_F is the force, which act on the wave-tool from the flux. And the axis x_1 coincides with the direction of the flux motion relatively of wave-tool. Now we shall project the components described by the formulas (25) and (26) on an axis x'' and y'', for summarizing its with components (13) and (14) of force, acting on the wave-tool from the continuum. For each of component (25) and (26) we obtain two projections. As it is visible from figure 6,

$$F_{x1x} = F_{x1}\cos\theta''; \qquad F_{x1y} = F_{x1}\sin\theta'';$$

$$F_{y1x} = -F_{y1}\sin\theta''; \quad F_{y1y} = F_{y1}\cos\theta''. \qquad (27)$$

If we insert in relations (8) the expressions (22) we shall receive:

$$\sin\theta'' = \frac{\beta_F \sin\theta\sqrt{1-\beta_T^2}}{\sqrt{\beta_F^2+\beta_T^2-2\beta_F\beta_T\cos\theta-\beta_F^2\beta_T^2\sin^2\theta}}, \quad (28)$$

$$\cos\theta'' = -\frac{\beta_F\cos\theta-\beta_T}{\sqrt{\beta_F^2+\beta_T^2-2\beta_F\beta_T\cos\theta-\beta_F^2\beta_T^2\sin^2\theta}}. (29)$$

The projections on axis x" and y" of force acting from the flux on the wave-tool in its own reference frame:

$$F''_{xF} = F_{x1x} + F_{y1x}, . \qquad (30)$$

$$F''_{yF} = F_{x1y} + F_{y1y}. \qquad (31)$$

By inserting the expressions (25), (26), (28), (29) in the formulas (27), and the results obtained in (30) and (31) we shall receive:

$$F''_{xF} = -\frac{\rho_T}{\rho_0} dSp_0\sin\theta, \qquad (32)$$

$$F''_{yF} = -\frac{\rho_T}{\rho_0} dSp_0\gamma_T(\beta_F\beta_T-\cos\theta). \qquad (33)$$

We copy expressions (13) and (14) for the component of forces acting from the continuum, taking into account, that φ=θ:

$$F_{Cx}'' = \frac{\rho}{\rho_0} dSp_0\sin\theta, \qquad (34)$$

$$F_{Cy}'' = -\frac{\rho}{\rho_0} dSp_0\gamma_T\cos\theta. \qquad (35)$$

Aggregate forces acting on the layer with thickness dS along the axis x and y in wave-tool frame:

$$F_x'' = F_{Cx}'' + F_{xF}''$$

$$F_y'' = F_{Cy}'' + F_{Fy}''.$$

By inserting in these formulas the expressions (32), (33) and (34), (35), we shall receive:

$$F_x'' = 0 \qquad \text{and} \qquad F_y'' = -\frac{\rho_T}{\rho_0} dSp_0\gamma_T\beta_F\beta_T.$$

Thus, the force, which act on the wave-tool **along the axis x, is equal to zero. Hence, on the wave-tool intersecting flux border, always act only the force, perpendicular to direction of the wave-tool motion:**

$$F'' = F_y'' = -\frac{\rho_T}{\rho_0} dSp_0\gamma_T\beta_F\beta_T \qquad (36)$$

Now we pass from wave-tool frame to laboratory system. Let's designate force, which act on the layer dS of the wave-tool on flux border in laboratory system as dF. This force will be equal:

$$dF = \frac{F''}{\gamma_T} = -\frac{\rho_T}{\rho_0} dSp_0\beta_F\beta_T.$$

To calculate force, which act on all wave-tool on border is necessary to substitute dS by S (figure 8). Where S the sectional area of wave-tool, which coincides with border.

$$F = -\frac{\rho_T}{\rho_0} p_0 S\beta_F\beta_T.$$

If the difference of velocity smoothly varying and is described by derivative from velocity on spatial coordinate l, then the force, which act in the domain of velocity difference will be:

$$F = -\frac{1}{c}\left(\frac{\rho_T}{\rho_0}V\right)\left(\frac{p_0}{c}\frac{dv_F}{dl}\right)v_T. \qquad (37)$$

As it follows from the deduction accomplish above, the vector of force F is located in same plane with the vector of flux velocity v_F and vector of wave-tool velocity v_T and is perpendicular to velocity of wave-tool.

If to designate:

$$q = \frac{\rho_T}{\rho_0} S(x_1 - x_2) = \frac{\rho_T}{\rho_0} S\Delta x = \frac{\rho_T}{\rho_0} V,$$

and $$B = \frac{p_0}{c}\frac{\partial v_F}{\partial l},$$

the formula (37) becomes identical to the formula, for Lorentz force which act on the electron, moving in the magnetic field.

Thus, it is possible to make a conclusion that, in the field of velocity variation of medium on the wave act the force similar to the force, acting on electrical charge in the magnetic field. The sign of force is determined by surplus or deficit of continuum carried by wave. Is remarkable, that all deductions are precise, that in physics happens seldom.

Bibliography

1. Mironov B. *On the metric of waves interactions. Meridian ingineresc. Nr.2, 2001, p. 54-59.*

2. Mironov B.*On the interaction of waves with inhomogeneities in the ideal continuous medium. Meridian ingineresc. Nr.4, 2003, p. 36-41.*

Recommended for publication: 09.09.2012.