TUNING OF CONTROLLERS TO THE THIRD ORDER ADVANCE DELAY OBJECTS WITH NONMINIMAL PHASE AND ASTATISM

Ion FIODOROV, Bartolomeu IZVOREANU, Irina COJUHARI, Svetlana TONU

Technical University of Moldova, Automatica Department,
Ştefan cel Mare av., 168, MD 2004, Chişinău, Republic of Moldova
Phone: (+373-22) 319175, fax: (+373-22) 319204
E-mail: izvor@mail.utm.md

Abstract: A procedure of tuning of linear controllers to the third order advance delay objects' models with nonminimal phase and astatism is proposed in this paper. The object is presented with transfer function that contains one positive zero and three negative poles. The maximal stability method is used for tuning of linear controllers. The tuning procedure of controllers represents an algebraic method with a reduce number of calculations. The results obtained on tuning of controllers through the selected method are compared with the results obtained in conformity of the known methods.

Key words: the advance delay object's model with nonminimal phase, the tuning of controllers, the maximal stability degree method.

INTRODUCTION

At automation of diverse technological processes the object's models are represented as advance delay models with respectively degree and nonminimal phase [1, 2, 3]. In this paper the object's model is represented by the following transfer function:

$$H(s) = \frac{k(1 - T_1 s)}{s(T_2 s + 1)(T_3 s + 1)(T_4 s + 1)} = \frac{b_1 - b_0 s}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s}; \qquad (1)$$

where k is the transfer coefficient, T_1, T_2, T_3, T_4 – time constants, the coefficients $b_1=k$; $b_0=kT_1$; $a_0=T_4T_3T_2$; $a_1= T_2T_3+T_2T_4+T_3T_4$; $a_2= T_2+T_3+T_4$; $a_3=1$. The transfer function (1) contains one positive zero, one negative pole and zero pole.

For the tuning of regulators to the class of objects' models (1) can be used known methods [1, 2, 3]. Many calculations and graphical representations follow frequency method. The Ziegler-Nichols method can't be used. The parametrical optimisation method is followed by even a greater number of calculations. To solve the difficulties indicated above, in this paper, the maximal stability degree method is proposed for tuning of P, PI, PID types of linear regulators for the class of objects' models represented in the form (1).

The maximal stability degree method was used in the paper [4] for tuning of regulators to the third order advance delay objects' models with nonminimal phase.

TUNING ALGORITHM OF CONTROLLERS

Assume that the structural scheme of control system is made of object with transfer function that has the form (1) and P, PI, PID controllers:

$$A_1(p)=\frac{a_0p^4+a_1p^3+a_2p^2+a_3p}{b_1-b_0p}+k_p=0, \tag{2}$$

$$A_2(p)=\frac{a_0p^5+a_1p^4+a_2p^3+a_3p^2}{b_1-b_0p}+k_p p+k_i=0, \tag{3}$$

$$A_3(p)=\frac{a_0p^5+a_1p^4+a_2p^3+a_3p^2}{b_1-b_0p}+k_d p^2+k_p p+k_i=0, \tag{4}$$

where k_d, k_i, k_p, are the tuning parameters of regulators P, I, D respectively and p – derivative operator. The expressions (2)..(4) are rewritten using the substitution $p=-J$:

$$A_1(-J)=\frac{a_0J^4-a_1J^3+a_2J^2-a_3J}{b_1+b_0J}+k_p=0, \tag{5}$$

$$A_2(-J)=\frac{-a_0J^5+a_1J^4-a_2J^3+a_3J^2}{b_1+b_0J}-k_pJ+k_i=0, \tag{6}$$

$$A_3(-J)=\frac{-a_0J^5+a_1J^4-a_2J^3+a_3J^2}{b_1+b_0J}+k_dJ^2-k_pJ+k_i=0, \tag{7}$$

where J is the maximal stability degree of the designed control system [2].

From the expression (5)..(7) we take the first, second and third order derivatives, in conformity with the number of tuning parameters of respectively regulator, and obtain the algebraic expressions which allow to determine the values of the maximal stability degrees of the designed control system.

The expressions for determination of tuning parameters of respectively regulators P, PI, PID after some transformations have the form:

- for P controller from (5):

$$k_p=\frac{-a_0J^4+a_1J^3-a_2J^2+a_3J}{b_1+b_0J}, \tag{8}$$

- for PI controller:

$$k_p=\frac{-c_0J^5+c_1J^4+c_2J^3-c_3J^2+c_4J}{(b_1+b_0J)^2}, \tag{9}$$

where $c_0=4a_0b_0$; $c_1=-5a_0b_1+3a_1b_0$; $c_2=4a_1b_1-2a_2b_0$; $c_3=3a_2b_1-a_3b_0$; $c_4=2a_3b_1$;

$$k_i = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2}{b_1 + b_0 J} + k_p J \ , \tag{10}$$

- for PID controller:

$$k_d = \frac{c_0^{'} J^6 - c_1^{'} J^5 + c_2^{'} J^4 - c_3^{'} J^3 - c_4^{'} J^2 + c_5^{'} J - c_6^{'}}{2(b_1^2 + 2b_0 b_1 J + b_0^2 J^2)^2} , \tag{11}$$

where $c_0^{'} = 3b_0^2 c_0 = 12a_0 b_0^3; c_1^{'} = -8c_0 b_0 b_1 + 2c_1 b_0^2 = -42a_0 b_0^2 b_1 + 6a_1 b_0^3;$
$c_2^{'} = 5c_0 b_1^2 - 6c_1 b_0 b_1 - c_2 b_0^2 = 50a_0 b_0 b_1^2 - 22a_0 b_1 b_0^2 + 2a_2 b_0^3;$
$c_3^{'} = 4c_1 b_1^2 + 4c_2 b_0 b_1 = -20a_0 b_1^3 + 28a_1 b_0 b_1^2 - 8a_2 b_0^2 b_1;$
$c_4^{'} = 3c_2 b_1^2 - 2c_3 b_0 b_1 - b_0^2 c_4 = 12a_1 b_1^3 - 12a_2 b_0 b_1^2; c_5^{'} = 2c_3 b_1^2 = 6a_2 b_1^3 - 2a_3 b_0 b_1^2;$
$c_6^{'} = c_4 b_1^2 = 2a_3 b_1^3;$

$$k_p = \frac{-c_0 J^5 + c_1 J^4 + c_2 J^3 - c_3 J^2 + c_4 J}{(b_1 + b_0 J)^2} + 2k_d J \ , \tag{12}$$

In (11) and (12) the coefficients c_0, c_1, c_2, c_3, c_4 have been used from (9).

$$k_i = \frac{a_0 J^5 - a_1 J^4 + a_2 J^3 - a_3 J^2}{b_1 + b_0 J} - k_d J^2 + k_p J \ . \tag{13}$$

With these calculations the tuning procedure of regulators to the models' objects form (1) with given parameters is over.

Apparently the determination of optimum stability degree from expressions (5), (6), (7) is the difficult procedure. The solution of this problem, in this paper, is proposed in following mode. The algebraic expressions (8)-(13) represent dependence functions of tuning parameters k_p, k_i, k_d of respective controllers P, PI, PID on unknown variable of maximal stability degree *J*. These dependencies k_p, k_i, $k_d = f(J)$ can be built and analysed.

APPLICATIONS AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning of the typical controllers let's examine an example with the object's model (1) which has the following parameters:

$$k = 3; T_1 = 0{,}8; T_2 = 0{,}5; T_3 = 1{,}6; T_4 = 1{,}2;$$

It is required to tune the P, PI, PID type of controllers. Doing the respectively calculations in conformity with the proposed method for the given object we obtained the following dependencies of tuning parameters of controllers on the variation of the maximal stability degree *J* of designed system figures 1, 2, 3, (curves *a*):

- for the control system (CS) with P controller:

1) J_1 =0.2; k_{p1} =0.0267334486; 2) J_2=0.3; $k_{p\,2}$ =0.0234446922; 3) J_3 =0.1; $k_{p\,3}$ =0.020406497;

a)

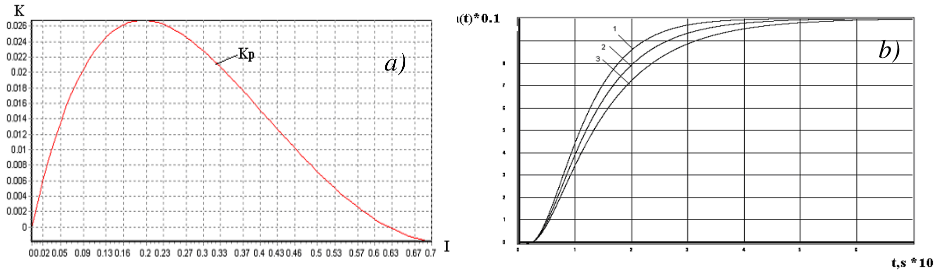

Fig.1.

Dependence of tuning parameter (*a*) and transition processes of CS (*b*)

- for the control system with PI controller:

1) J_1 = 0.12; $k_{p\,1}$ =0.0342332201; $k_{i\,1}$ = 0.0012606180;
2) J_2 =0.18; $k_{p\,2}$ = 0.0291785774; $k_{i\,2}$ =0.00045662976;
3) J_3 =0.06; $k_{p\,3}$ =0.02701336726; $k_{i\,3}$ =0.00068903337;

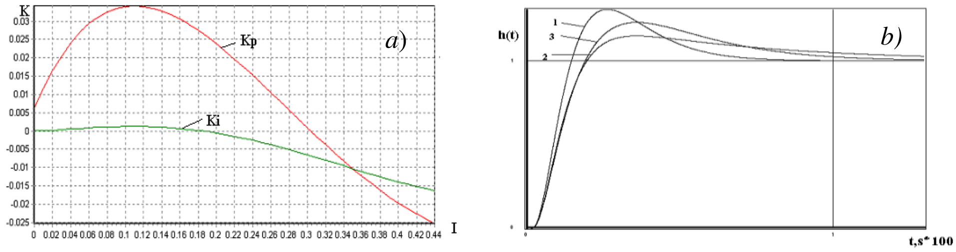

Fig.2. Dependencies of tuning parameters (*a*) and transition processes of CS (*b*)

- for the control system with PID controller:

1) J_1 = 0.3; $k_{p\,1}$ =0.075289259; $k_{i\,1}$=0.004942980; $k_{d\,1}$=0.119993362;
2) J_2 = 0.45; $k_{p\,2}$ = 0.0321684712; $k_{i\,2}$ = 0.003426852; $k_{d\,2}$ = 0.0639524531;
3) J_3 =0.15; $k_{p\,3}$ =0.0422074008; $k_{i\,3}$ = 0.0017787693; $k_{d\,3}$ =0.0307405594;

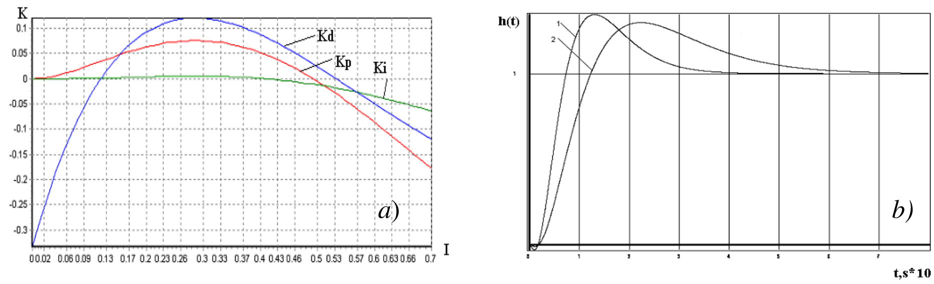

Fig.3. Dependencies of tuning parameters (*a*) and transition processes of CS (*b*)

On the computer was simulated the control system with the given objects'models forms (1) and respectively controllers P, PI, PID. The results of computer simulations are represented in the figures 1, 2, 3, (curves *b*).

CONCLUSIONS

As a result of the study, which was made for given class of objects models, the following conclusion can be made:

1. The proposed tuning algorithm for linear controllers P, PI, PID to given third order advance delay objects'models with nonminimal phase and astatism (1) represents an algebraic method.

2. The proposed tuning algorithm represents a simple procedure which consists of following stages:

- the value of optimum stability degree of the designed system with respectively type of controller is determined from the dependencies k_p, k_i, k_d=$f(J)$;
- the tuning parameters of respectively regulators are determined from algebraic expressions.

REFERENCES

[1] Preitl, Ş., Precup, R.-E. (2001) *Introducere în ingineria reglării automate,* Timişoara: Editura Politehnica.

[2] Zagarii,G.I., Shubladze, A.M. (1988) *Sintez sistem upravlenia na osnove criteria maximalinoi stepeni ustoicivosti (The Synthesis of the Control System According to the Maximal Stability Degree),* Moskva: Energatomizdat.

[3] *Sintez reguleatorov sistem avtomaticeskogo upravlenia / Pod red. K. A. Pupkova i N. D. Egupova. (*2004), Moskva: Izd-vo MGTU, V. 3, 616str.

[4] Izvoreanu, B., Fiodorov, I., Pisarenco M. (2004) *Tuning of Linear Regulators to the Third and Fourth Order Advance Delay Models of Objects with Nonminimal Phase / Proceedings of the 7th International Conference on Development and Application Systems*, Suceava, p. 37-42.