POWERFUL TURBO REED-SOLOMON CODES FOR WIRELESS COMMUNICATIONS

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ABSTRACT

Wireless communications are vulnerable to Additive White Gaussian Noise (AWGN) and fading phenomenon [1]. Random errors or error-bursts occur. A Reed-Solomon (RS) code defined on a Galois Field (GF) has a good error-correction capacity but the Turbo-RS (TRS) code, also known as product code, is better than the simple one [2]. Different high-rate TRS codes are designed in this paper and their performances are analyzed and compared.

Key words: wireless communication, error-burst, error-correction, Reed-Solomon code, turbo code.

INTRODUCTION

The principle of turbo coding based on a block code is illustrated in figure 1. A square data matrix of k^2 symbols obtained from the data-vector is coded row-by-row and column-by-column, using the basic Reed-Solomon RS (n, k) code defined on a Galois Field GF (2^m), and the square Turbo Reed-Solomon (TRS) coded matrix with n^2 elements results. Three parity matrices are deduced for every input-block (P_H - horizontal parity; P_V - vertical parity; P_A - auxiliary parity).

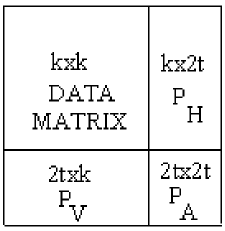

Fig. 1 Turbo-RS(k+2t,k) Coding Structure

The coding rate of the symmetric TRS (n, k) based is given by:

$$R = \frac{k^2}{n^2} \tag{1}$$

The maximum number of correctable symbol-errors t by a RS (n, k) code is equal to:

$t = \frac{n-k}{2}$ [Symbols] (2)

A TRS (*n, k*) code, defined on GF (2^m) can correct a maximum error-burst of about:

$S = t \cdot n$ [Consecutive symbols] (3)

$B = m \cdot (S-2) + 2 = m \cdot (nt-2) + 2$ [Consecutive bits] (4)

The systematic RS (*n, k*) code and the associated TRS (*n, k*) could be described by the generator matrix $G = [I|P]$, (I – the identity matrix, P – the puncturing matrix).

The RS code word results as (all the arithmetical operations are defined on the GF):

$\vec{c} = \vec{a} \cdot G$ (5)

($\vec{a}$ is the input vector of *k* symbols; $\vec{c}$ is the code word of *n* symbols).

The TRS decoder applies the error-correcting RS algorithm row-by-row and column-by-column, for a limited number of iterations. It detects the transmission errors when the n-points Discrete-Fourier Transform (n-DFT) of the received codeword has not the imposed 2*t* zeroes on the last positions:

$\vec{R} = n - DFT\{\vec{r}\}; R_{n-k} + R_{n-k+1} + ... + R_{n-1} > 0$ (6)

The n-DFT of the error-vector ($\vec{E} = n - DFT\{\vec{e}\}$) is computed using the error locator polynomial L(x), with the maximum degree *t*, which verifies:

$L(x) \cdot E(x) \bmod ulo(x^n) = 0, E_i = R_i, i = \overline{n-k, n-1}$ (7)

The error-vector and the corrected vector results:

$\vec{e} = n - IDFT\{\vec{E}\}$ (8)

$\vec{d} = \vec{r} + \vec{e}$ (9)

For a systematic code, data are recovered directly from the received sequence, so no decoder is needed.

The upper bound of the block-error rate BLER is expressed as [2]:

$BLER \le \sum_h D_h H^h \Big|_{H=\exp(-RE_s/2N_0)}$ (10)

E_s / N_0 is the signal-to-noise ratio (SNR). D_h is the number of code words with weight *h*.

For high dimensions codes the upper bound of BLER has a great number of terms. A good approximation results if only the first terms, starting with the minimum *h,* are computed.

The coding gain is computed based on the minimum non-zero code word weight:

$G(dB) = 10 * \log(h_m R)$ (11)

EXPERIMENTAL

Different Turbo-RS coding/decoding Matlab algorithms are designed (Table I).

Table I. Turbo Reed-Solomon Codes Parameters

GF symbol dimension	Turbo Code Name	Codeword length	Input-word length	TRS Code Parameters	
m (bits)		n^2 (symbols)	k^2 (symbols)	***S*** (symbols)	***B*** (bits)
3	TRS (7,5)	49	25	7	17
4	TRS (15,11)	225	121	30	114
5	TRS (31,27)	961	729	62	302

The encoding algorithms were implemented based on relation (5). The decoding algorithms are more complex for *t* greater than 1 when more error locator polynomials are considered, for all possible number of errors, from 1 to *t*. Maximum 10 turbo-decoding iterations are made.

RESULTS

The transposed puncturing matrices of RS (7, 5) and RS (15, 11) codes are deduced as:

$$P75^T = \begin{bmatrix} 6 & 7 & 7 & 1 & 6 \\ 2 & 2 & 3 & 1 & 3 \end{bmatrix}.$$

$$P1511^T = \begin{bmatrix} 6 & 13 & 14 & 12 & 7 & 13 & 8 & 7 & 2 & 2 & 13 \\ 8 & 8 & 7 & 13 & 9 & 9 & 15 & 13 & 14 & 11 & 12 \\ 14 & 11 & 12 & 8 & 15 & 10 & 3 & 13 & 6 & 5 & 8 \\ 5 & 12 & 2 & 6 & 5 & 5 & 6 & 14 & 14 & 15 & 7 \end{bmatrix}.$$

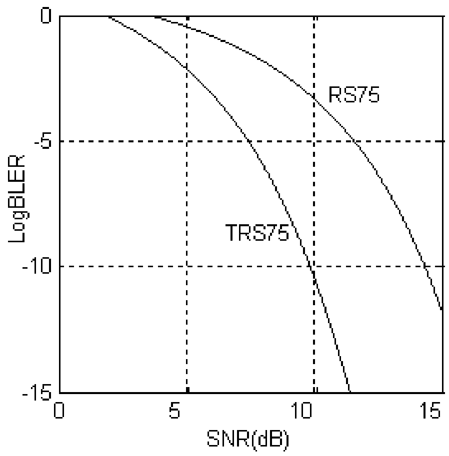

Figure 2. RS(7, 5) and TRS(7, 5) BLER

Similarly the puncturing matrix of RS (31, 27) code was calculated.

Some of the non-zero D_h coefficients of the RS and TRS codes were deduced for some values of weight *h*. For example:

RS (7, 5): $D_0 = 1, D_2 = 4, D_3 = 8, D_4 = 3, D_5 = 4, D_6 = 5, D_7 = 4, D_8 = 7, D_9 = 4.$

TRS (7, 5): $D_0 = 1, D_9 = 59, D_{12} = 67, D_{16} = 66, D_{17} = 47.$

The coding gain values of these codes are:

$G_{RS75} = 1.5490\ dB$; $G_{TRS75} = 6.6199\ dB$

The RS (7,5) and the TRS (7, 5) decoders BLER are plotted depending on SNR.

DISCUSSION

The coding algorithm is implemented easier using the puncturing matrix included into the systematic generator matrix of the code. The TRS decoding algorithm, based on the RS decoder, is working iteratively with a limited number of iteration. A compromise between the error-correction performances and the decoding time is made. The TRS code increases the coding gain substantially and decrease the BLER in comparison to the basic RS code.

CONCLUSIONS

For higher transmission rates an increased error-block length results and powerful TRS codes must be used to ensure a good quality of communication process for low SNR. The TRS code could be implemented software, by algorithms, or hardware, in embedded communication systems. Fast algorithms for DFT and IDFT computation on GF have to be used for high-speed communications and real-time applications.

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