

# Polarization-Singular Processing of Biological Layers Laser Images in Order to Diagnose and Classify their Optical Properties

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**Abstract** – Presented in this work are the results of investigations aimed at analysis of coordinate distributions of the fourth Stokes vector parameter in laser images of three types of phase-inhomogeneous layers, namely: rough, ground and bulk scattering layers. To characterize this parameter for all the types of phase-inhomogeneous layers, the authors have offered to use three groups of parameters: statistic moments of the first to fourth orders, autocorrelation functions, logarithmic dependences for power spectra. Ascertained are the criteria for diagnostics and classification of phase-inhomogeneous layers optical properties.

**Index Terms** – polarization, singularity, birefringence, autocorrelation, Stokes vector, Jones matrix.

## I. INTRODUCTION

By tradition, the processes of transforming optical radiation of phase-inhomogeneous objects and media are considered, as a rule, in a statistic approach (theory of radiation transfer [1], Monte-Carlo modeling [2]). Among the most spread traditional methods for studying the scattered light fields, one can separate the following independent directions: “scalar” (photometry and spectrophotometry) [3, 4] and “vector” (polarization nephelometry, Mueller-matrix optics) [5 - 16]. Using these approaches, determined are interrelations between the sets of statistic moments of the 1-st to the 4-th orders [6, 7, 11, 15], correlation functions [5, 8, 9, 14], fractal dimensions [5-7] that characterize phase-inhomogeneous or rough surfaces and coordinate distributions for phases [15, 16], azimuths and ellipticity of polarization in their laser images [6 - 16].

In parallel with traditional statistic investigations, formed in recent 10 to 15 years is the new optical approach to describe a structure of polarizationally inhomogeneous fields in the case of scattered coherent radiation. The main feature of this approach is the analysis of definite polarization states to determine the whole structure of coordinate distributions for azimuths and ellipticities of polarization. The so-called polarization singularities are commonly used as these states [15, 17, 18]:

- states with linear polarization when the direction of rotation for the electric field vector is indefinite, the so-called L-points;
- circularly-polarized states when the azimuth of polarization for the electric field vector is indefinite, the so-called C-points.

It is noteworthy that there exists a widespread group of optically anisotropic biological objects for which the methods of laser polarimetric diagnostics are not so efficient. Optically-thin (coefficient of extinction  $\tau \leq 0.1$ ) layers of various biological fluids (bile, urine, liquor, synovial fluid, blood plasma, etc.) can be related to these objects. All these layers possess considerably less optical anisotropy (the possibility of C-points forming is sufficiently small) of the biological component matter as compared with birefringent biological tissue structures [5]. On the other hand, the biological fluids are more available for a direct laboratory analysis as compared to traumatic methods of biological

tissues biopsy. From the above reasoning, it seems topical to search new, additional parameters for laser diagnostics of optically anisotropic structures in biological fluids.

This work is aimed at ascertaining the possibilities to diagnose and classify phase-inhomogeneous layers (PhIL) of various types (surface-scattering, subsurface-scattering and bulk-scattering ones) by determination values and ranges for changing the statistic (moments of the 1-st to the 4-th orders), correlation (autocorrelation functions) and fractal (logarithmic dependences for power spectra) parameters that characterize coordinate distributions for polarization-singular states in PhIL laser images.

## II. MODEL CONCEPTION

As a base for analytical description of processes providing formation of polarization-inhomogeneous images for various types of PhIL, we have used the model conceptions developed in the works [5-8]:

- surface-scattering PhIL is a rough surface (superficial layer of the skin epithelium) consisting of an ensemble of quasi-plane, chaotically oriented micro-areas with optical dimensions  $l > \lambda$  - **group 1**;
- PhIL with surface and subsurface scattering – ground glass with rough external and subsurface (the layer of collagen fibrils of the skin derma) components - **group 2**;
- PhIL with bulk scattering – optically thick layer of the skin derma of a various optical thickness - **group 3**.

Mechanisms providing formation of polarization-inhomogeneous images for rough surface

Optical properties of each micro-area of rough layer of the epithelium are exhaustively characterized with the Jones operator of the following look

$$\{R\} = \begin{pmatrix} 1 & 0 \\ 0 & p_y/p_x \end{pmatrix}. \quad (1)$$

It is possible to show that within the sizes  $(\Delta x, \Delta y)$  of one micro-area there takes place the change of polarization azimuth  $\alpha$  inherent to the refracted plane-polarized laser wave with the initial azimuth  $\alpha_0$

$$\alpha(\Delta x, \Delta y) = \arctg\left(\frac{p_y U_{0y}}{p_x U_{0x}}\right) = \arctg\left[\left(\Delta p_{xy}\right)_{fg} \alpha_0\right] \quad (2)$$

where  $U_{0x}$ ,  $U_{0y}$  are orthogonal components of the amplitude  $U_0$ ,  $p_x$ ,  $p_y$  - Fresnel amplitude coefficients for transmission [5].

Thus, in the approach of single scattering the polarization image of rough surface may be considered as coordinate-distributed parts of  $L$ -polarized states [9, 10].

Model structure of PhIL with surface and subsurface components – ground surfaces

The process providing formation of a local polarization state can be considered as superposition of “influences” of an optically strained subsurface of optically anisotropic layer of collagen fibrils as well as the surface rough micro-relief one disposed in sequence. From the analytical viewpoint, this scenario can be described by superposition  $\{F\}$  of the Jones matrix operators for these partial layers (subsurface  $\{T\}$  and surface  $\{R\}$ )

$$\{F\} = \{R\}\{T\} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \begin{bmatrix} (r_{11}t_{11} + r_{12}t_{21}) & (r_{11}t_{12} + r_{12}t_{22}) \\ (r_{21}t_{11} + r_{22}t_{21}) & (r_{21}t_{12} + r_{22}t_{22}) \end{bmatrix}, \quad (3)$$

$$\{T\} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} \cos^2 \gamma + \sin^2 \gamma \exp(i\delta) & \cos \gamma \sin \gamma \exp(i\delta) \\ \cos \gamma \sin \gamma \exp(i\delta) & \sin^2 \gamma + \cos^2 \gamma \exp(i\delta) \end{bmatrix}. \quad (4)$$

Here,  $\gamma$  is the direction of the optical axis of fibril;  $\delta$  - phase shift between orthogonal components ( $U_x$ ,  $U_y$ ) of the amplitude ( $U$ ) of laser wave with the wavelength  $\lambda$  that arises as a consequence of birefringence in the matter  $\Delta n$ .

If taking into account the relations (1), (3) and (4), it follows that within the limits  $(\Delta x, \Delta y)$  of a local bulk formed is an elliptically polarized part of the object field with the following parameters

$$\tilde{\alpha}(\Delta x, \Delta y) = \arccos \left( \frac{\sin \delta}{\cos 2 \left( \arctg \left\{ \left[ \frac{(f_{21} + f_{22})^2}{(f_{11} + f_{12})^2} \right] \text{tg } \alpha_0 \right\} \right)} \right) \quad (5)$$

$$\tilde{\beta}(\Delta x, \Delta y) = \arcsin \left( \frac{\text{tg } \delta}{\sin 2 \left( \arctg \left\{ \left[ \frac{(f_{21} + f_{22})^2}{(f_{11} + f_{12})^2} \right] \text{tg } \alpha_0 \right\} \right)} \right) \quad (6)$$

As it follows from the analytical relations (5) and (6), interaction of the plane-polarized ( $\alpha_0$ ) wave with the PhIL of this type provides formation of a polarization-inhomogeneous laser image. Among the whole set of values  $(\tilde{\alpha}, \tilde{\beta})$ , formation of  $L$  and  $\pm C$  polarization states seems to be very probable [7, 15]

$$L- \Leftrightarrow \delta(\Delta x, \Delta y) = q\pi, \quad q = 1, 2, \dots \quad (7)$$

$$\pm C- \Leftrightarrow \text{tg } \delta(\Delta x, \Delta y) =$$

$$\sin 2 \left( \arctg \left\{ \left[ \frac{(f_{21}(\Delta x, \Delta y) + f_{22}(\Delta x, \Delta y))^2}{(f_{11}(\Delta x, \Delta y) + f_{12}(\Delta x, \Delta y))^2} \right] \text{tg } \alpha_0 \right\} \right) \quad (8)$$

Polarization structure of laser fields inherent to PhIL with bulk scattering

When analyzing the processes of interaction of laser radiation with these PhIL, we have used the method of superposition of the Jones matrix operators (3) for the set of sequentially disposed optically-thin layers

$$\{\Phi\} = \{\Phi^{(p)}\} \{\Phi^{(p-1)}\} \times \dots \times \{\Phi^{(1)}\} \quad (9)$$

Having calculated the set of Jones matrix elements  $\phi_{qg}$  for an optically-thick PhIL, one can define analytical expressions (like to (7) – (8)) to find  $L$  and  $\pm C$  polarization states in the laser image

$$L- \Leftrightarrow \delta^*(\Delta x, \Delta y) = q\pi, \quad q = 1, 2, \dots \quad (10)$$

$$\pm C- \Leftrightarrow \text{tg } \delta^*(\Delta x, \Delta y) =$$

$$\sin 2 \left( \arctg \left\{ \left[ \frac{(\phi_{21}(\Delta x, \Delta y) + \phi_{22}(\Delta x, \Delta y))^2}{(\phi_{11}(\Delta x, \Delta y) + \phi_{12}(\Delta x, \Delta y))^2} \right] \text{tg } \alpha_0 \right\} \right) \quad (11)$$

Thus, the above analytical consideration (relations (1) to (11)) for various scenarios of transformation of laser radiation by PhIL in all the cases enabled to reveal the principled possibility of formation of polarization-singular states ( $\beta = 0$ ,  $\beta = \pm \pi/4$ ) in respective laser images.

In this work, to describe coordinate  $(x, y)$  distributions for polarization-singular ( $L$ ,  $\pm C$ ) states in laser images for all the types of PhIL [7, 15]

$$\begin{cases} V_4(x, y) = 0 \Leftrightarrow L(\beta = 0); \\ V_4(x, y) = \pm 1 \Leftrightarrow \pm C(\beta = \pm \pi/4) \end{cases} \quad (12)$$

### III. EXPERIMENTAL SETUP

Our study of polarization-inhomogeneous laser images inherent to PhIL was performed using the optical scheme of a laser polarimeter (figure 1) [5, 15]

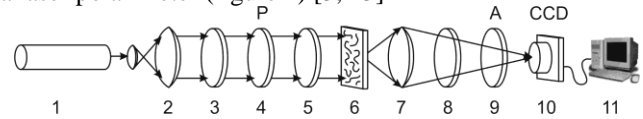


Fig. 1. Optical scheme of the polarimeter: 1 – He-Ne laser; 2 – collimator; 3, 5, 8 – quarter-wave plates; 4, 9 – polarizer and analyzer, respectively; 6 – object under investigation; 7 – micro-objective; 10 – CCD camera; 11 – personal computer

Illumination was performed using a parallel beam ( $\varnothing = 10^4 \mu\text{m}$ ) from a He-Ne laser ( $\lambda = 0.6328 \mu\text{m}$ ) 1. The polarization illuminator (quarter-wave plates 3 and 5 as well as polarizer 4) were used to form various polarization states in the laser beam. Polarization images of PhIL 6 were projected using the micro-objective 7 into the plane of the light-sensitive area ( $800 \text{pix} \times 600 \text{pix}$ ) in CCD-camera 10. Turning the transmission axis of the analyzer 9 by the angles  $\pm 45^\circ$  relatively to the direction of the highest velocity axis for the quarter-wave plate 8, we could determine the intensities of right ( $I_{\otimes}$ ) and left ( $I_{\oplus}$ ) circularly polarized components for each separated pixel of CCD camera 10. It served as a base to calculate coordinate distributions of the fourth parameter in the Stokes vector  $V_4(m \times n)$  describing the laser image of PhIL, if using the relation

$$V_4(r_{mn}) = \frac{I_{\otimes}(r_{ik}) - I_{\oplus}(r_{ik})}{I_{\otimes}(r_{ik}) + I_{\oplus}(r_{ik})}. \quad (13)$$

The two-dimensional array (13) was scanned along the horizontal direction  $x \equiv 1, \dots, m$  with the step  $\Delta x = 1 \text{pix}$ . Within the limits of each local sample  $(1_{\text{pix}} \times n_{\text{pix}})^{k=1, 2, \dots, m}$ , we calculated the amount ( $N$ ) of characteristic values  $V_4(k) = 0$ ,  $-(N_L^{(k)})$  and  $V_4(k) = \pm 1$ ,  $-(N_{\pm C}^{(k)})$ .

Thus, we determined the dependences  $N_L(x) \equiv (N_L^{(1)}, N_L^{(2)}, \dots, N_L^{(m)})$  and  $N_{\pm C}(x) \equiv (N_{\pm C}^{(1)}, N_{\pm C}^{(2)}, \dots, N_{\pm C}^{(m)})$  for amounts of polarization-singular  $L$ - and  $\pm C$ -points within the limits of a laser image for PhIL.

#### IV. ESTIMATION CRITERIA FOR POLARIZATION IMAGES OF PHIL

Distributions  $N_{L,\pm C}(x)$  for the amount of polarization-singular states in laser images of PhIL are characterized with the set of statistic moments of the 1-st to the 4-th orders  $Z_{j=1,2,3,4}$  calculated using the following relations [6, 7]

$$Z_1 = \frac{1}{M} \sum_{i=1}^M |N_{L,\pm C}^{(i)}(x)|, \quad Z_2 = \sqrt{\frac{1}{M} \sum_{i=1}^M [N_{L,\pm C}^{(i)}(x)]^2}, \quad (14)$$

$$Z_3 = \frac{1}{Z_2^3} \frac{1}{M} \sum_{i=1}^M [N_{L,\pm C}^{(i)}(x)]^3, \quad Z_4 = \frac{1}{Z_2^4} \frac{1}{M} \sum_{i=1}^M [N_{L,\pm C}^{(i)}(x)]^4.$$

where  $N = 800 \times 600$  is the amount of pixels in CCD camera 10 (Fig. 1).

Our analysis of the coordinate structure for  $N_{L,\pm C}(x)$  distributions was based on the autocorrelation method by using the function [15]

$$K_{L,\pm C}(m) = \frac{1}{(n-m)\sigma^2} \sum_{t=1}^{n-m} [X_t - \mu][X_{t+m} - \mu], \quad (15)$$

Here,  $n$  is the length of discrete sampling  $N_{L,\pm C}(x) = X_1, X_2, \dots, X_n$ ;  $\mu$  - average value,  $\sigma^2$  - the dispersion;  $m, n$  - positive integers; ( $m = 1 \text{ pix}$ ) is the step for changing the coordinate  $x = 1 \div m$ .

As correlation parameters that characterize the dependences  $K_{L,\pm C}(\Delta x)$ , we chose:

- correlation area  $S_{L,\pm C}$

$$S_{L,\pm C} = \int_1^m K_{L,\pm C}(m) dm, \quad (16)$$

- normalized fourth statistic moment  $Q_{L,\pm C}$  that determine the kurtosis of the autocorrelation function  $K_{L,\pm C}(m)$

$$Q_{L,\pm C} = \frac{N}{\left( \sum_{i=1}^N (K_{L,\pm C}(m))_i^2 \right)^2} \sum_{i=1}^N (K_{L,\pm C}(m))_i^4; \quad (17)$$

The fractal analysis of the distributions  $N_{L,\pm C}(x)$  was performed using the calculation of logarithmic dependences  $\log J[N_{L,\pm C}(x)] - \log d^{-1}$  for the power spectra  $J[N_{L,\pm C}(x)]$  which was calculated as a discrete Fourier transform of the corresponding autocorrelation function  $K_{L,\pm C}(m)$  using the MatLab software

$$J[N_{L,\pm C}(x)] = S_{xx}(\omega) = \sum_{m=1}^n K_{L,\pm C}(m) e^{-j\omega m}, \quad (18)$$

where  $\omega$  are the normalized frequencies, which correspond to a spatial frequencies ( $\omega = d^{-1}$ ) that are determined by geometrical sizes ( $d$ ) of PhIL structural elements.

The dependences  $\log J[N_{L,\pm C}(x)] - \log d^{-1}$  are approximated using the least-squares method into the curves  $\Phi(\eta)$ , straight parts of which serve to determine the slope angles  $\eta$  and calculate fractal  $F$  dimensions by using the relations [6, 15]

$$F_{L,\pm C} = 3 - \text{tg} \eta. \quad (19)$$

Classification of coordinate distributions  $N_{L,\pm C}(x)$  should be performed using the following criteria [14, 15]:

- they are fractal on the condition of a constant slope angle value  $\eta = \text{const}$  for 2 to 3 decades of changing sizes  $d$ ;
- they are multi-fractal, if several slope angles  $\Phi(\eta)$  are available;
- they are random when any stable slope angles are absent within the whole range of changing sizes  $d$ .

In the latter case, the distributions  $\log J[N_{L,\pm C}(x)] - \log d^{-1}$  are characterized with the dispersion

$$D_z = \sqrt{\frac{1}{m} \sum_{i=1}^m [\log J(N_{L,\pm C}(x_i)) - \log d^{-1}]^2}. \quad (20)$$

#### V. THE INVESTIGATION OBJECTS CHARACTERISTICS

Fig. 1 illustrates coordinate ( $100 \text{ pix} \times 50 \text{ pix}$ ) distributions of the fourth parameter for the Stokes vector  $V_4(m \times n)$  inherent to laser images of PhIL in all the groups.

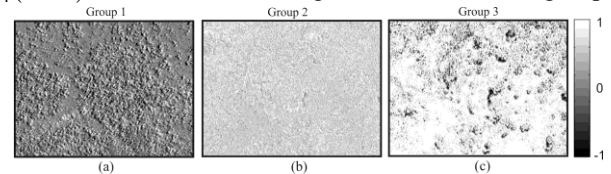


Fig. 1. Coordinate distributions of  $V_4(m \times n)$  of laser images inherent to PhIL

Our qualitative analysis of coordinate distributions  $V_4(m \times n)$  for laser images of PhIL (figure 1) enabled to reveal:

- Practically all the images of the rough surface of skin (figure 1a) are linearly polarized field  $V_4(m \times n) = 0$  (relations (1) and (2)). Availability of a small amount of the parts  $V_4(m \times n) \neq 0$  polarized otherwise can be related with interferential effects of multiple interaction of coherent waves with adjacent micro-roughnesses.
- The image of the rough skin surface with a subsurface layer of the derma (figure 1b) is characterized with a developed polarization-inhomogeneous structure formed both by linearly ( $V_4(m \times n) = 0$ ) and elliptically ( $V_4(m \times n) \neq 0$ ) polarized states, including the circularly ( $V_4(m \times n) = 1$ ) polarized ones (relations (5) to (8)).
- The images of the optically thick layer of skin (figure 1c) are characterized with the widest range of changing the azimuth and polarization due to multiple bulk scattering (relations (10-12)),  $-1 \leq V_4(m \times n) \leq 1$ .

VI. RESULTS

The performed analysis of results for statistic ( $Z_{j=1,2,3,4}^{L,\pm C}$ ), correlation ( $S^{L,\pm C}$ ,  $Q^{L,\pm C}$ ) and fractal ( $F^{L,\pm C}$ ,  $D^{L,\pm C}$ ) parameters has shown:

- *Statistic parameters.* The most sensitive appears to be both the 1<sup>st</sup> and the 2<sup>nd</sup> statistic moments, which characterize the distributions of  $L$ – polarization states, and the 3<sup>rd</sup> and the 4<sup>th</sup> statistic moments, which characterize the distributions of  $\pm C$ –polarization states. The difference between of them reaches 2-3 times for  $L$ –states and 5-7 times for  $\pm C$ –states;
- *Correlation parameters.* The most sensitive appears to be the normalized fourth statistic moment  $Q_{L,\pm C}$  that determine the kurtosis of the autocorrelation function  $K_{L,\pm C}(m)$ . The intergroup difference reaches one order of magnitude as for  $L$ –states as for  $\pm C$ –states;
- *Fractal parameters.* The fractal analysis appears to be effective in differentiation of optical properties of different PhIL too. The difference between the dispersion  $D^{\pm C}$  values reaches 2 times;

The possibility to differentiate “group” optical properties of PhIL with surface, subsurface and bulk light scattering is illustrated in Table 1.

TABLE I. THE DIFFERENTIATION POSSIBILITIES

PhIL Parameters	Groups 1 – 3	
	$N_L$	$N_{\pm C}$
$Z_1$	⊕	⊕
$Z_2$	⊕	⊕
$Z_3$	⊗	⊕
$Z_4$	⊗	⊕
$S$	⊗	⊕
$Q$	⊗	⊕
$F$	⊗	⊕
$D$	⊗	⊕

Note: ⊗ - here differentiation is impossible; ⊕ - possible.

VII. CONCLUSION

1. Analyzed in this work are the main physical mechanisms providing formation of polarization singularities in laser images of PhIL with surface, subsurface and bulk light scattering.
2. Offered are statistical, correlation and fractal parameters for polarization-singular estimating the optical properties inherent to PhIL of all types.
3. Determined are the ranges for changing the set of criteria that characterize distributions of the amount of polarization-singular states in laser images, which enabled us to realize both “intergroup” classification and differentiation of optical properties related to PhIL of various types.

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