

# THE ELABORATION OF MULTIPLE PRECESSIONAL GEAR THEORY AND MODERN MANUFACTURING TECHNOLOGY

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## INTRODUCTION

The diversity of beneficiaries' requirements concern mechanical transmissions is reduced, specially, to the increase of reliability, efficiency and to the decrease of mass and dimensions. It becomes more and more difficult to satisfy the mentions requirements by updating partially the traditional transmissions. This problem can be solved by using new types of mechanical transmissions-planetary precessional transmissions.

The engineering complex study of the triad "gear-technology-transmission" has permitted to elaborate a new type of precessional transmissions with multicouple gear, which, from the technological point of view can be manufactured via a new method of conical teeth with convex-concave profile processing.

The specific character of sphere-spatial (precessional motions of the precessional transmissions pinion makes impossible the utilisation of teeth classical involute profiles. This fact requires the elaboration of new profiles adequate to the sphere-spatial motion of pinion which would ensure high performances to the precessional transmission. Carrying out on the principle of the transfer function continuity and gear based on the principles of the transfer function continuity and gear multiplicity which aims to:

- the elaboration of the gear mathematics model with account of the peculiarities;
- the analytical description of teeth profiles by a system of parametric equations on spherical surface and normal teeth section for inner and plane gear;
- the determination by CAD of geometrical and cinematic parameters influence of the gear upon the teeth profiles shape and the justification of their rational limits of variation;
- the elaboration of the theoretical basis evaluation of teeth gear multiplicity in precessional transmissions;
- the definition of area of gear multiplicity existence by 100% teeth couples.

The production of non-standard teeth profiles requires a new manufacturing technology.

In the complexity of problem "gear-synthesis-profile study- manufacturing" an important role plays the elaboration of efficient methods of teeth manufacturing which ensures a maximum productivity and reduced cost while satisfying the requirements related to the gear with precessional motion. To solve this problem the following has been done:

- we elaborated the mathematics model of teeth generation which shows the interaction of teeth in precessional gear;
- we investigated the kinematics of the mechanism of method realisation for teeth generating;
- we determined the trajectory of the tool motion and the wrapping of the generating surface family of it by using the computer;
- we elaborated and manufactured from metal milling and tooth grinding tools, inclusively their longitudinal modification.

Constructions peculiarities and high multiplicity of gear create favourable premises for the improvement of precessional transmissions kinematics accuracy. Within these activities we elaborated:

- theoretical basis for the identification of kinematics error generated by various primary error (frontal and radial knocking), on the basis of error independent action principle by fulfilling computer assisted mathematics experiment;
- compensation method for manufacturing and assembling errors;
- method of determination of probable limit error for precessional reducers with account of the stochastic character of manufacturing and assembling errors.

Know-how in the elaboration of the multicouple precessional gear, manufacturing technology and control methods, and a range of precessional transmission diagrams belong to research team from the Technical University of Moldova. During the last 20 years the team patented about 125 inventions.

## 1. ANALYTICAL DESCRIPTION OF THE TEETH PROFILES

The engineering complex study of the triad "gear-technology-transmission" has permitted to elaborate a new type of precessional transmissions with multicouple gear, which, from the technological point of view, can be manufactured via a new method of conical teeth with convex-concave profile processing.

In precessional transmission the gear wheel produces sphere-spatial motion round a fixed point. It is known [1,2,3], that the body, which produces spherical motion, has three degrees of freedom. As a rule, in theoretical mechanics, the position of the body, which produces precessional motion, is determined by Euler angles. In this case, the mobile system of co-ordinates  $OX_1Y_1Z_1$  is **bound rigidly with the gear wheel, as origins of** the system of co-ordinates the immobile point  $O$  (centre of precession) being chosen (figure1).

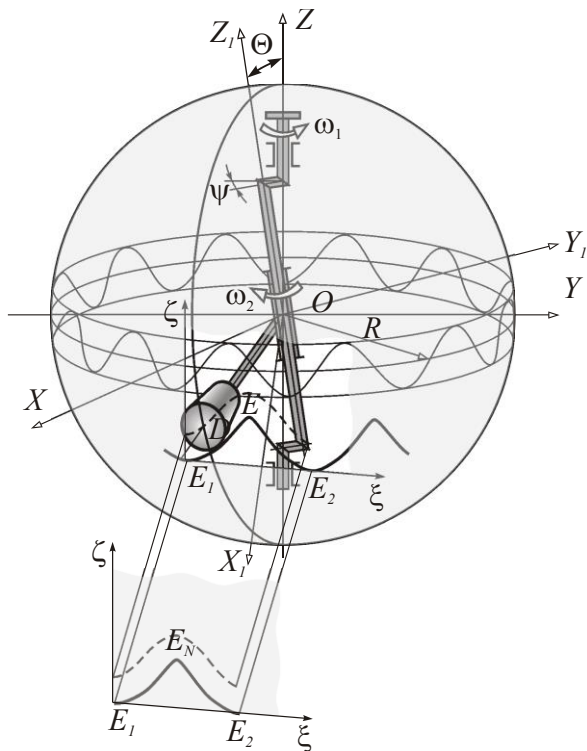


Figure 1. Profile teeth's determination.

This system of co-ordinates produces jointly with the gear wheel spherical motion related to the immobile system of coordinates  $OXYZ$ . Analytical relationship between the co-ordinates of gear wheel points, shown in the mobile  $OX_1Y_1Z_1$  and immobile  $OXYZ$  system of co-ordinates, was obtained analysis of the two pinion positions - initial one, when the

system of coordinates axis coincide, and final one, when the axis al removed (displaced).

By expressing vectors  $\bar{i}'_1, \bar{j}'_1, \bar{k}'_1$  via basic vectors  $\bar{i}, \bar{j}, \bar{k}$  of the immobile system of coordinates  $OXYZ$  we obtain

$$\begin{aligned} \bar{i}'_1 &= \bar{i} \cos \psi + \bar{j} \sin \psi, \\ \bar{j}'_1 &= -\bar{i} \sin \psi + \bar{j} \cos \psi, \quad \bar{k}'_1 = \bar{k} \end{aligned} \quad (1)$$

The second rotation is produced at angle  $\theta$  ( $0 \leq \theta \leq \pi$ ) round joint lines, after which the unit vector  $\bar{i}'_1, \bar{j}'_1, \bar{k}'_1$  will move into directions  $\bar{i}''_1, \bar{j}''_1, \bar{k}''_1$  (respectively, they coincide with the directions of axis  $OX'', OY'', OZ''$ ), at the same time vector  $\bar{k}''_1$ , which coincide with vector  $\bar{k}'_1$ , defines the position of axis  $OZ_1$  in final position.

By expressing vectors  $\bar{i}''_1, \bar{j}''_1, \bar{k}''_1$  via  $\bar{i}'_1, \bar{j}'_1, \bar{k}'_1$  we obtain

$$\begin{aligned} \bar{i}''_1 &= \bar{i}'_1; \\ \bar{j}''_1 &= -\bar{j}'_1 \cos \theta + \bar{k}'_1 \sin \theta; \\ \bar{k}''_1 &= -\bar{j}'_1 \sin \theta + \bar{k}'_1 \cos \theta. \end{aligned} \quad (2)$$

By operating matrix theory, transition from gear wheel point coordinates is produced (for example, of the roller centre  $D$ ), given in the mobile system of co-ordinates  $OX_1Y_1Z_1$  to the coordinates of the same points in the immobile system  $OXYZ$ . After some modifications we obtain

$$\begin{aligned} X_D &= R \cos \delta [-\cos \psi \sin(Z_1/Z_2) + \sin \psi \cos(Z_1\psi/Z_2)] - \\ &\quad - R \sin \delta \sin \psi \sin \theta; \\ Y_D &= -R \cos \delta [\sin \psi \sin(Z_1/Z_2) + \cos \psi \cos(Z_1\psi/Z_2)] - \\ &\quad - R \sin \delta \cos \psi \sin \theta; \\ Z_D &= -R \cos \delta \cos(Z_1\psi/Z_2) \sin \theta - R \sin \delta \cos \theta. \end{aligned} \quad (3)$$

Point  $D$  moves at the spherical surface by radius  $R$  with its centre in the precessional centre  $O$  (figure 1). Being familiar with the trajectory of roller centre motion, the position of the contact point by central wheel tooth is determined, which family in a precessional cycle represent the shape of the wheel tooth. After some modifications we obtain

$$\xi = X_E^n \cos \frac{\pi}{Z_1} + [R_D \cos(\delta + \beta) + Y_E^n] \sin \frac{\pi}{Z_1};$$

$$\zeta = X_E^n \sin \gamma \sin \frac{\pi}{Z_1} - [R_D \cos(\delta + \beta) + Y_E^n \cos \frac{\pi}{Z_1} + [R_D \sin(\delta + \beta) + Z_E^n] \cos \gamma.] \quad (4)$$

were

$$\sin \gamma = \operatorname{tg}(\delta + \beta) \sqrt{\left[ \cos^2 \frac{\pi}{Z_1} + \operatorname{tg}(\delta + \beta) \right]^{1/2}};$$

$$\cos \gamma = \cos \frac{\pi}{Z_1} \sqrt{\left[ \cos^2 \frac{\pi}{Z_1} + \operatorname{tg}(\delta + \beta) \right]^{1/2}}.$$

In figure 2 the profillograms of teeth, obtained for the various geometrical parameters of gear are shown. The analyses of profillograms demonstrate the lever and direction of influence upon the angle of conical axoid  $\delta$ , medium radius of well  $R_m$ , rollers radius  $R$ , conical angle of rollers  $\beta$  and correlation between number of teeth  $Z_1$  and  $Z_2$ .

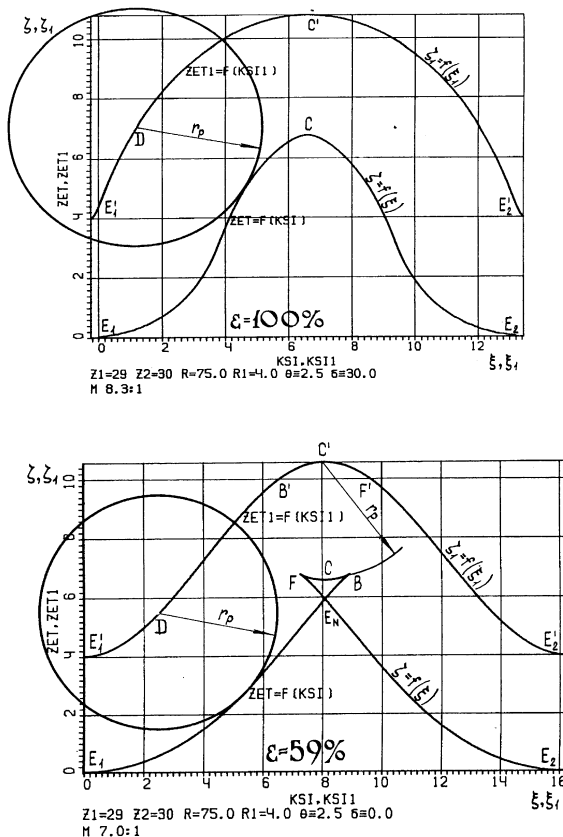


Figure 2. The profillograms of teeth with convex-concave profile.

### 3. KINEMATICS OF THE REALIZING MECHANISM FOR THE TEETH GENERATING METHOD

To realise the method of teeth processing [1,4,5] we have elaborated the mechanism. In the elaborated mechanism (figure 3) the node, which involves the tool into precessional motion, is fixed not to rotate round the common axis of the principal shaft - semiproduct shaft with a binding mechanism.

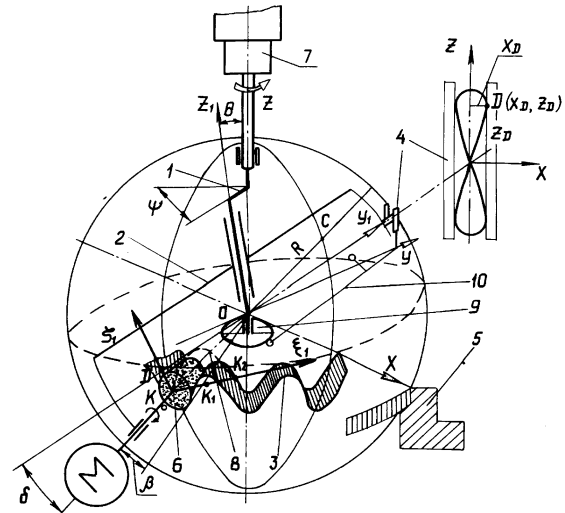


Figure 3. Spatial scheme of the realizing for the teeth processing method by rolling using precessional tool.

The constructive execution of the tool binding mechanism with the shaft ensures the continuity of the transmission function  $\omega_1/\omega_3 = \text{const.}$  and is determined by the motion trajectory of point  $C$  which belongs to the movable part. Setting up the position function of the binding mechanism and the motion equation of the generating wheel and using the matrix device for transferring from the movable system of coordinates  $X_1 Y_1 Z_1$  to the immovable one  $OXYZ$ , we determined the coordinates of point  $C$

$$\begin{aligned} X_c &= R_c (1 - \cos \theta) \cos \psi \sin \psi; \\ Y_c &= R_c (\sin^2 \psi + \cos \theta \cos^2 \psi); \\ Z_c &= R_c \sin \theta \sin \psi. \end{aligned} \quad (5)$$

These equations (5) represent the parametric equations of the supporting surface of the binding mechanisms, inserted in the device.

#### 4. DETERMINATION OF THE TOOL MOTION TRAJECTORY

For the angle of the conical axoid of the teeth wheel  $\delta = 0$  the equation of tool motion are identical to the equation (4), having only opposite values. In the case of toothed wheels processing with an angle of the conical axoid  $\delta > 0$  the centre of the tool will have the co-ordinates in the movable system  $OX_1Y_1Z_1$   $X_{1D} = 0; Y_{1D} = -R \cos \delta;$   
 $Z_{1D} = -R \sin \delta.$

Then the equation of tool motion in the immovable system of coordinates  $OXYZ$  will be

$$\begin{aligned} X_D &= -R_U \cos(1 - \cos \theta) \cos \psi \sin \psi - \\ &- R_U \sin \delta \sin \psi; \\ Y_D &= -R_U \cos \delta (\sin^2 \psi - \cos \theta \cos^2 \psi) + \\ &+ R_U \sin \delta \cos \psi; \\ Z_D &= -R_U \cos \delta \cos \psi - R_U \sin \delta. \end{aligned} \quad (6)$$

#### 5. DETERMINATION OF THE FAMILY WRAPPING OF TOOL SURFACES

The wrapping is determined by the equations of the working surface of the generating tool and by the relative motion parameters while wrapping. To make easier the determination of wrapping we pass to the tool centre co-ordinates in the movable system of co-ordinates, bound to the semiproduct

$$\begin{aligned} \bar{X} &= X_D \cos \psi_3 + Y_D \sin \psi_3; \\ \bar{Y} &= -X_D \sin \psi_3 + Y_D \cos \psi_3; \\ \bar{Z} &= Z_D. \end{aligned} \quad (7)$$

where  $\bar{X}_D, \bar{Y}_D, \bar{Z}_D$  are the coordinates of the tool centre in the movable system of co-ordinates;  $\psi_3 = \psi/U$  is the rotation angle of the semiproduct;  $U$  is the transmission ratio of the cinematic chain "principal shaft - semiproduct".

The equations (7) define the motion trajectory of tool centre, evaluated on the sphere. Then we determined the equation of wrapping on the sphere (figure 4). The analyses of profillograms demonstrate the lever and direction of influence upon the angle of tool insertion  $\delta$  by the rotation axis of the semiproduct, tool radius  $R$  and transmission ratio  $U$  of the cinematic chain

"principal shaft - semiproduct" on processed tooth profile.

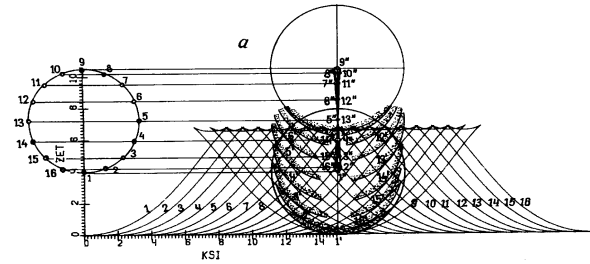


Figure 4. The family wrapping of tool surfaces.

The elaboration of working machines driving mechanisms is based on the diagram of precessional transmissions, presented in figure 5. The rotating motion of the crank shaft 1 is transformed into sphere-spatial motion of the block pinion 2 with two teathed crowns 6 and 7, which are rolling without sliding on the immovable and driven toothed wheel teeth 3 and 4. Due to the minimum difference between the number of teeth  $Z_3 = Z_6 - 1, Z_4 = Z_7 - 1$ ,  $Z_6 = Z_7 - 1, 2, 3, \dots$  the transmission ratio is

$$U = \pm \frac{Z_6 Z_4}{Z_3 Z_7 - Z_6 Z_4}.$$

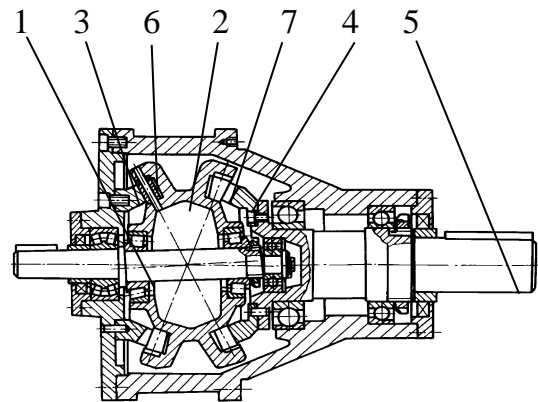


Figure 5. Planetary precessional transmission.

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