METHODOLOGY TO EVALUATE THE PROFILE EQUATION FOR A CAM TYPE SPARE PART

Dr. Paraschiv, E. Merticaru, V. Merticaru jr.
Technical University "Gh. Asachi" of Iassy

1. GENERAL CONSIDERATIONS

The majority of the automatic machines, which carry out multiple technological operations, contain in their structure cam mechanisms, through which are realized the command movements and the driving of the final elements. The activities of repairing and machining the spare parts for the cam type pieces require knowing the follower motion law.

In the present paper a methodology for determining the motion law machined on an existing cam is proposed. This methodology is exemplified for the case of a plane mechanism with rotating disk cam and oscillating roller follower.

The methodology presented in the paper can be used when it is not possible to measure directly the follower's displacement depending on the cam rotation angle.

2. THE ANALYZED MECHANISM

The plain mechanism on which is exemplified the methodology for determining the motion law of the cam, contains a grooved cam and is part of the driving cinematic chain of the thread picker from a loom type STB-216.

The cinematic schema of the mentioned mechanism is presented in figure 1, where the dimensions are as following:
AD = d = 69 mm; DC = l = 60,7 mm; R = 8 mm;
R = follower roller's radium; DE = 185,1 mm;
CE = 125 mm; EH = 215,5 mm; FG = 13,75 mm;
D = 120 mm; D = cam diameter;
x_D = -34,07 mm; y_D = -60 mm;
x_G = 71,5 mm; y_G = 110,9 mm; AG = 132 mm;
Minimum radium of the cam's real profile:
$r_{min.real}$ = 24 mm;
Maximum radium of the cam's real profile:
$r_{max.real}$ = 40,428 mm.
The functioning range of the number of rotations per minute, for the cam, is: n = 180 ÷ 240 rot/min.
So, the angular speed for the cam is in the range:

$$\omega_1 = \frac{\pi \cdot n}{30} = 18{,}85 \div 25{,}13 \text{ rad/s.}$$

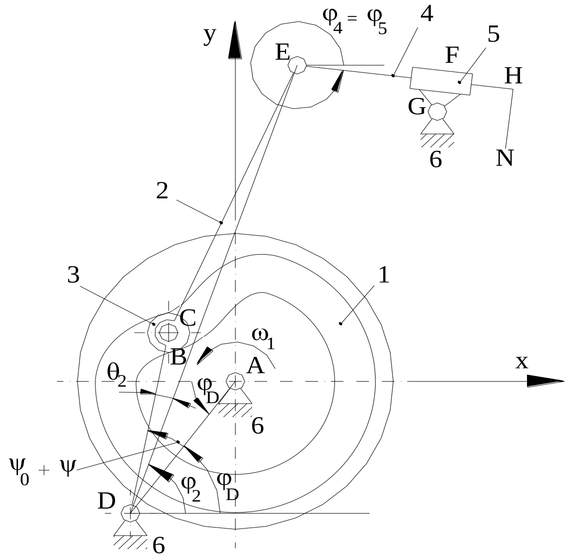

Figure 1. The schema of the analyzed mechanism.

3. METHODOLOGY TO EVALUATE THE CAM'S PROFILE

In a first stage it is necessary to measure the cam profile, that is, to measure the radius of the cam profile in function of the position angle β of the radius (the polar coordinates of the points on the cam profile).

A comparative device was used for measuring the cam's profile. The cam, fixed on a mandrel, was rotated, successively, with an angular step of 1º, by using a dividing head.

The measured values r of the radius on the cam's profile would be corrected by considering the pick-up radium, in the sense of determining the real values for the radium on the cam's profile.

The correction that considers the pick-up radium, applied to the measured values r, was realized considering: R_p = pick-up radium; S_i = values of the pick-up displacement; r_i = the measured values r of the cam profile's radius.

Calculus of the correction for the radium values will be made as for a cam mechanism with translation follower. The formulas are:

$$R_p = 1{,}05 \text{ mm} \; ; \qquad (1)$$

$$r_i = rr_{\min} + S_i \text{ [mm]}, \tag{2}$$

where: rr_{min} = minimum radium of the cam's real profile = 24 mm ;

$$r_{ti} = rr_{\min} + R_p + S_i = r_i + R_p \text{ [mm]}, \tag{3}$$

The corrected real radium values of the cam profile will be calculated by formula:

$$rr_i = \sqrt{r_{ti}^2 + R_p^2 - 2 \cdot r_{ti} \cdot R_p \cdot \cos\alpha_i} \text{ [mm]}, \tag{4}$$

and the value α_i of the pressure angle is defined by relation:

$$\alpha_i = arctg\left(\frac{k_f \cdot \frac{dS_i}{d\beta_i}}{r_{ti}}\right) . \tag{5}$$

In relation (5): k_f = phase coefficient, having, respectively, the values (+1) or (-1), as the ratio $\left(\frac{v}{\omega_1}\right)$ is positive or negative (where v represents the translation follower's speed).

In the same relation (5), $\frac{dS_i}{d\beta_i}$ was calculated by the approximate formula:

$$\frac{dS_i}{d\beta_i} \cong \frac{S_i - S_{i-1}}{\beta_i - \beta_{i-1}} . \tag{6}$$

The radial error (or the radium correction) can be calculated by formula:

$$er_i = rr_i - r_i . \tag{7}$$

The correction for the angle β, due to the radium of the pick-up rod , is defined by relation:

$$e\beta_i = \arcsin\left(R_p \cdot \frac{\sin\alpha_i}{rr_i}\right). \tag{8}$$

So, the real (corrected) value of the angle β will be calculated with formula:

$$\beta_{ri} = \beta_i + e\beta_i . \tag{9}$$

For calculating the values of the corrections er_i, $e\beta_i$ and the corrected radius and angular values, rr and β_r, a computerized calculation program "*corectie*", in TurboPascal, was especially conceived. The obtained values for er_i are presented in figure 2. The corresponding obtained values $e\beta_i$ are presented in figure 3. The real radium values rr and the real angular values β_r , which both represent the polar coordinates of the cam's profile, are presented in figure 4.

The polar coordinates of the cam's profile (rr and β_r) will be used for calculating the following parameters:

- ψ_0 = the minimum value of the angle ∠ADC ;
- ψ_a = the follower stroke;
- φ_u = the rise stroke angle;
- φ_{ss} = the superior dwell period angle;
- φ_c = the fall stroke angle;
- φ_{si} = the inferior dwell period angle;
- the motion law for the rise stroke and for the fall stroke.

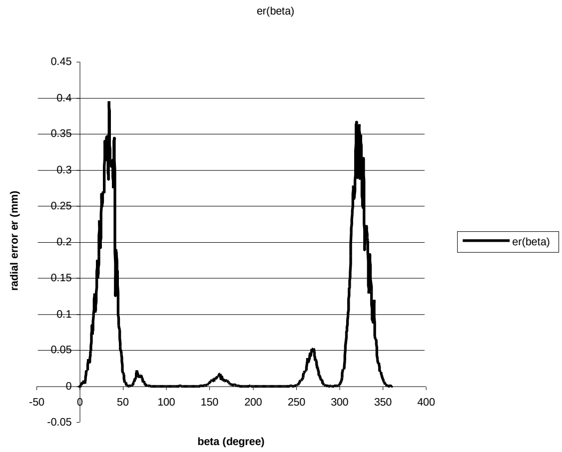

Figure 2. The obtained values for er_i.

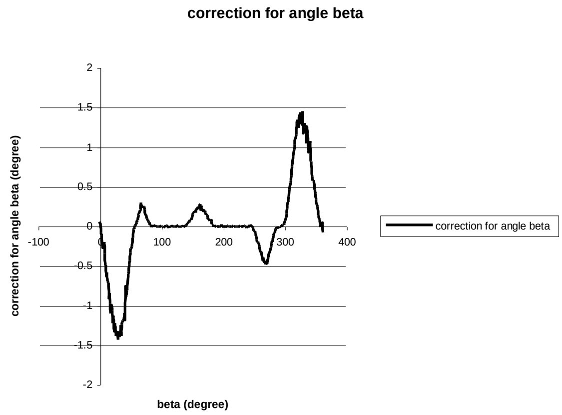

Figure 3. The obtained values $e\beta_i$.

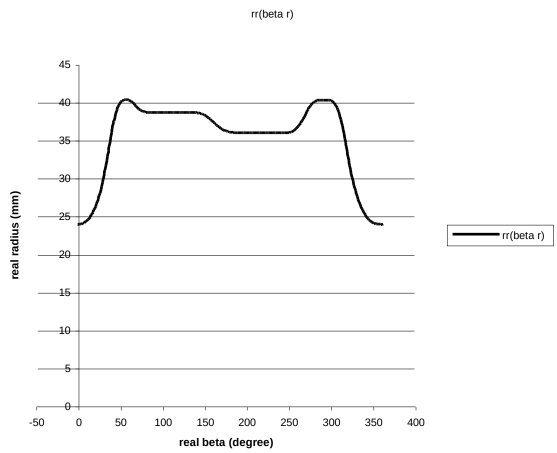

Figure 4. The real radium values rr and the real angular values β_r

Calculus is based on the local approximation of the cam's profile by a number of circles containing, each of them, three successive points on the profile.

For the rise stroke, calculus schemas are shown in figure 5, where the notations represent:

$\delta_0 = \angle B_0AD_0$; $\delta = \angle BAD$; $\varphi = \angle D_0AD$; $\beta = \angle B_0AB$;

$AB_0 = r_{\min}$ =minimum radium on the real profile;

$AB = r$ = radium for the point B, on the real profile;

$B_0C_0 = BC = R$ = radium of the follower roller;

$DC = l$ = follower's length;

$AD = d$ = distance between the cam's axis and the follower's axis.

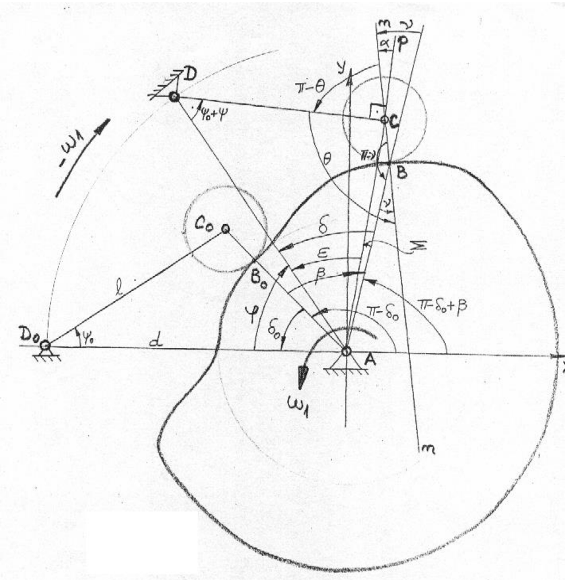

Figure 5. Calculus schema

There are considered known the values: R, l, d, and the pairs of values (β, r) for each point considered on the real profile of the cam. The value of the angle δ_0 is defined by relation:

$$\delta_0 = \arccos\left[\frac{(r_{\min} + R)^2 + d^2 - l^2}{2 \cdot (r_{\min} + R) \cdot d}\right]. \tag{10}$$

The Cartesian coordinates of an i ordinated point on the cam's profile, are calculated by formulas:

$$\begin{cases} x_i = x_B = r_i \cdot \cos(\pi - \delta_0 + \beta_i) \\ y_i = y_B = r_i \cdot \sin(\pi - \delta_0 + \beta_i) \end{cases}. \tag{11}$$

In figure 6, three successive points C, B, A, of (i-1), (i) and respectively (i+1) ordination, are considered on the cam's profile. Between these points, the cam's profile is approximated by a circle centered in point M and containing the points C, B, A. The coordinates of the points P and N, which are the midpoints on the segments CB and respectively BA, are defined by relations:

$$x_P = \frac{x_i + x_{i-1}}{2}\ ;\ y_P = \frac{y_i + y_{i-1}}{2}\ ; \tag{12}$$

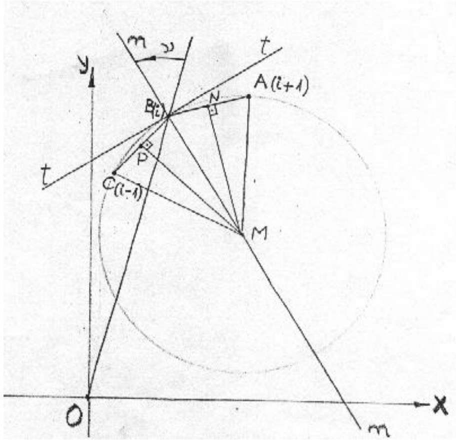

Figure 6. Calculus schema.

$$x_N = \frac{x_{i+1} + x_i}{2}\ ;\ y_N = \frac{y_{i+1} + y_i}{2}\ ; \tag{13}$$

The equation of the line (CB) is:

$$y - y_C = m_1 \cdot (x - x_C) \quad \text{or}$$

$$y - y_{i-1} = m_1 \cdot (x - x_{i-1}) \tag{14}$$

$$\text{where}: \ m_1 = \frac{y_B - y_C}{x_B - x_C} = \frac{y_i - y_{i-1}}{x_i - x_{i-1}}\ . \tag{15}$$

The equation of line (MP), perpendicular on (CB):

$$y - y_P = m_2 \cdot (x - x_P), \tag{16}$$

$$\text{where:} \ m_2 = -\frac{1}{m_1} = -\frac{x_B - x_C}{y_B - y_C}\ . \tag{17}$$

The equation of the line (BA) is:

$$y - y_B = m_3 \cdot (x - x_B)\ , \tag{18}$$

$$\text{where}: \quad m_3 = \frac{y_A - y_B}{x_A - x_B}\ . \tag{19}$$

The equation of line (MN), perpendicular on (BA):

$$y - y_N = m_4 \cdot (x - x_N)\ , \tag{20}$$

$$\text{where:} \ m_4 = -\frac{1}{m_3} = -\frac{x_A - x_B}{y_A - y_B}\ . \tag{21}$$

At the intersection of the lines (MP) and (MN), there is defined the point M, so the coordinates of this point, which is the curvature center for the profile in point B, can be determined by solving the system containing the equations (16) and (20), respectively:

$$\begin{cases} y - y_P = m_2 \cdot (x - x_P) \\ y - y_N = m_4 \cdot (x - x_N) \end{cases}. \tag{22}$$

The coordinates of the point M are:

$$\begin{cases} x_M = \dfrac{m_2 \cdot x_P - y_P - m_4 \cdot x_N + y_N}{m_2 - m_4} \\ y_M = m_2 \cdot x_M - m_2 \cdot x_P + y_P \end{cases} \tag{23}$$

where: $x_B = x_i$; $y_B = y_i$; $x_C = x_{i-1}$; $y_C = y_{i-1}$; $x_A = x_{i+1}$; $y_A = y_{i+1}$.

The equation of line nn perpendicular to line (MB):

$$y - y_M = m_5 \cdot (x - x_M), \tag{24}$$

where: $$m_5 = \frac{y_B - y_M}{x_B - x_M}. \tag{25}$$

The equation of the line OB (the position vector of the point B from the cam's profile) is:

$$y = m_6 \cdot x, \tag{26}$$

where: $$m_6 = \frac{y_B}{x_B}. \tag{27}$$

The angle ν formed by the position vector of one point from the cam's profile and the perpendicular *nn* on the profile, respectively ν = ∠(OB;MB), is:

$$tg\nu = \frac{m_5 - m_6}{1 + m_5 \cdot m_6}. \tag{28}$$

The curvature radius of the real profile is:

$$\rho = \frac{\left(\sqrt{\left(\frac{dx_i}{dh}\right)^2 + \left(\frac{dy_i}{dh}\right)^2}\right)^3}{\left(\frac{dx_i}{dh}\right)\cdot\left(\frac{d^2 y_i}{dh^2}\right) - \left(\frac{d^2 x_i}{dh^2}\right)\cdot\left(\frac{dy_i}{dh}\right)}, \tag{29}$$

where: $h = \beta_i - \beta_{i-1}$; $\frac{dx_i}{dh} \cong \frac{x_i - x_{i-1}}{h}$;

$\frac{d^2 x_i}{dh^2} \cong \frac{x_{i+1} - 2\cdot x_i + x_{i-1}}{h^2}$; $\frac{dy_i}{dh} \cong \frac{y_i - y_{i-1}}{h}$;

$$\frac{d^2 y_i}{dh^2} \cong \frac{y_{i+1} - 2\cdot y_i + y_{i-1}}{h^2}. \tag{30}$$

The curvature radius for the theoretical profile is:

$$\rho_t = \rho + R. \tag{31}$$

So, for calculating the radius of the theoretical profile, it can be used the formula:

$$AC^2 = BC^2 + AB^2 - 2\cdot BC\cdot AB\cdot\cos(\pi - \nu) \tag{32}$$

or: $$r_t^2 = R^2 + r^2 - 2\cdot R\cdot r\cdot\cos(\pi - \nu). \tag{33}$$

The value of the angle ψ_0 will be:

$$\psi_0 = \arccos\left(\frac{l^2 + d^2 - r_{t.\min}^2}{2\cdot l\cdot d}\right) \tag{34}$$

and the value of the angle ψ will be:

$$\psi = \arccos\left(\frac{l^2 + d^2 - r_t^2}{2\cdot l\cdot d}\right) - \psi_0. \tag{35}$$

The follower stroke ψ_a is defined by relation:

$$\psi_a = \arccos\left(\frac{l^2 + d^2 - r_{t.\max}^2}{2\cdot l\cdot d}\right) - \psi_0. \tag{36}$$

The angle φ of the cam rotation will be:

$$\varphi = \delta - \delta_0 + \beta, \tag{37}$$

where the angle δ is: $\delta = \Sigma + \varepsilon$, (38)

with: Σ = ∠BAC ; ε = ∠CAD (fig. 5).

The angles Σ and ε will be determined by relations:

$$\varepsilon = \arccos\left(\frac{r_t^2 + d^2 - l^2}{2\cdot d\cdot r_t}\right); \tag{39}$$

$$\Sigma = \arcsin\left(\frac{R\cdot\sin(\pi - \nu)}{r_t}\right). \tag{40}$$

The rise stroke angle, φ_u, will be calculated:

$$\varphi_u = \left|\delta_u - \delta_0 + \beta_u\right|, \tag{41}$$

where: δ_u and β_u represent the values of the angles δ and β at the end of the rise stroke.

The pressure angle α (fig. 5) will be calculated:

$$\alpha = \theta - \frac{\pi}{2}, \tag{42}$$

where: θ = ∠DCB (fig. 5), with:

$$\theta = \pi - \psi_0 - \psi + \nu - \delta. \tag{43}$$

For the calculus of the parameters ψ_0, ψ_a, φ_u, φ_{ss}, φ_c, φ_{si} and for determining the follower's motion law, corresponding to the rise stroke and to the fall stroke, described by figures 5 and 6 and by relations (10)÷(43), there was especially conceived, in TurboPascal, the computerized calculation program "*recam2u*"(for the rise stroke).

For the fall stroke, the calculus is similar and the program conceived in this sense is named "*recam2c*".

Also for the dwell periods, there can be used, for calculus, the program "*recam2u*" or the program "*recam2c*".

4. CONCLUSIONS

A methodology for determining the motion law machined on an existing cam was presented. This methodology is exemplified for a plane mechanism with rotating disk cam and oscillating roller follower.

The methodology presented in the paper can be used when it is not possible to measure directly the follower's displacement depending on the cam rotation angle.

The results of the paper are useful for the activities of repairing and machining the spare parts for the cam type pieces, which require to know the follower motion law.

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