Birefringence in Quantum Wells

Dorogan A., Dorogan V., Syrbu N.
Technical University of Moldova,
Chisinau, Republic of Moldova

Zalamai V.
Institute of Applied Physics,
Academy of Sciences of Moldova,
Chisinau, Republic of Moldova

***Abstract*—Polarization dependences of reflection and wavelength modulated reflection spectra of quantum wells $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ were investigated. Spectral dependences of refractive indices, extinction coefficients, real and imaginary parts of dielectric constants quantum wells structures for different polarizations were calculated by the Kramers-Kronig analysis. A phenomenon of birefringence and an interference of polarized light waves in quantum wells were researched. The isotropic wavelength was found out (λ_0 = 1.246 μm). Interference spectra changes the density of fringes and refractive indices intersect zero axis at energy of isotropic wavelength (0.955 eV).**

***Index Terms*—Quantum wells; birefringence; Reflection spectra; Dispersion equation; Kramers-Kronig analysis; Isotropic point; Optical functions;**

I. Introduction

Quantum well (QW) heterojunctions are used in many domains of optoelectronics and optical communication. The variety of optoelectronic devices as different types of lasers, radiation detectors, modulators, optical amplifiers, commutators etc. on the base of QW was developed. The investigation of semiconductors heterostructures with QW is relevant from the point of view of identifying the physical processes and parameters in QWs. Due to quantum-mechanical mixing of light and heavy holes states the optical transitions between states with QWs are very sensitive to the light polarization. Structures with the periodical set of WQs are similar with the uniaxial crystal from the point of view of its optical properties. The anisotropy of optical properties in QW layers is also observed. The birefringence and gyration are discovered in such structures. The nonideality of structure influences on optical spectra of reflection, transmission and luminescence, it leads to the nonuniform broadening of the quantum transition resonance frequency (ω_0) in heterostructures. This nonuniform can lead to the gradual coordinate dependence of ω_0 in the plain of QW or in the volume of superlattice, that causes the broadening of absorption and transmission lines.

This work dedicated to investigations of reflection spectra for different polarizations of QW heterojunctions $In_{0.68}Al_{0.1}Ga_{0.13}As/In_{0.42}Al_{0.22}Ga_{0.24}As/InP$. Calculations of reflection spectra contours were executed using of dispersion equations (two-oscillator model) and Kramers-Kronig integrals. The anisotropy of optical functions due to birefringence and the isotropic wavelength were observed. It was shown, that effective masses of heavy and light holes are differ to each other in two times.

II. Experimental methods

Optical spectra of reflection and transmission are measured on spectrometers MDR-2 and Jasco-670 at temperature 300 K for S- and P- polarizations (P ~ E||Z, S ~ E⊥Z) and different incidence angles of light on the surface of hetero-nanostructure. If it is necessary the fine spectral structure was measures using high-aperture (1:2), double Raman spectrometer SDL-1 with high resolution. The back side of some samples was polished as a mirror for transmission spectra measurements. Linear polarizers and analyzers for optical polarization measurements are made from the iceland spar and oriented relative to the each other and QWs grow axis. Three types of samples with the same composition only with different technological parameters of molecular beam epitaxial process were researched.

III. Experimental results and discussions

Fig. 1 shows reflection spectra of QW $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ heterostructures grown on InP substrate. The thickness of QW is equal to 6 nm and the thickness of barrier layer is 7 nm (on the insert of Fig. 1 is presented structure investigated samples). Reflection spectra (marked on Fig. 1 as α, β and γ) correspond to layers with the same composition but grown with different technological conditions. Spectral dependences of reflection coefficient of heterostructures with QWs were analyzed by a lot of authors (see for example Ref. [9 - 11]) and the reflection of QW (r_{QW}) is defined by equation:

$$r_{QW} = \frac{i\Gamma_0}{\omega_0 - \omega - i(\Gamma + \Gamma_0)}; \quad (1)$$

$\omega'_o = \omega_o + r_{10}\Gamma_o \sin 2\varphi$, $\Gamma_o = \Gamma_o (1 + r_{10}\cos 2\varphi)$.

where ω'_o is resonance frequency and Γ_o is radiation damping of quantum transition. The spectral dependence of reflection coefficient is determined by next expression:

$$R(\omega) = |r(\omega)|^2 = R_0 + (A+Bx)/(1+x^2); \quad (2)$$

where $x = (\omega - \omega_0)/\Gamma$, $R_0 = r_{01}^2$;

$$A = t_{01}\, t_{10}\, S\, [t_{01}\, t_{10}\, S - 2\, r_{01}\, (1 + S) \cos 2\varphi]; \quad (3)$$

$$B = 2\, r_{01}\, t_{01}\, t_{10}\, S \sin 2\varphi,\ S = \Gamma_0/\Gamma. \quad (4)$$

Taking into account Fresnel formulas at normal incidence of light on the crystal surface has reflectivity:

$$r_{10} = - r_{01} = (n_b - 1)/(n_b + 1),\ t_{01}\, t_{10} = 4\, n_b/(n_b + 1)^2. \quad (5)$$

TABLE 1: STRUCTURE AND LAYERS THICKNESSES OF INVESTIGATED SAMPLES.

Composition	Layer	Thickness, nm
InP	InP-surface	20
$In_{0.42}Al_{0.22}Ga_{0.24}As$	Br	7
$In_{0.68}Al_{0.1}Ga_{0.13}As$	QW	6
$In_{0.42}Al_{0.22}Ga_{0.24}As$	Br	7
InP	In substrate	350

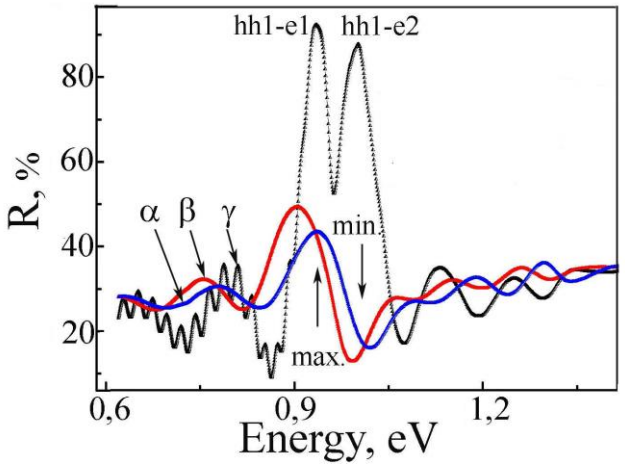

Fig. 1. Reflection spectra of QW heterostructure $In_{0.68}Al_{0.1}Ga_{0.13}As/In_{0.42}Al_{0.22}Ga_{0.24}As$ grown on InP substrate. Reflection spectra α, β and γ correspond to samples taken from different technological groups.

Coefficients A and B can take on values of different signs in dependence of the distance between well center and outer surface and specifically its can become zero by turns. The resonance contour consists of the maximum at $\omega < \omega'_0$ and the minimum at $\omega > \omega'_0$ in the case then A = 0 and B < 0. At B = 0 the one maximum (A > 0) or one minimum (A < 0) is present in the spectrum [9 - 11]. Reflectivity spectra of investigating nano-heterostructures $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ (Fig. 1, curves α, β and γ) correspond to the case A = 0 and B < 0. The nonidealness of structure has an influence on optical spectra of reflection and absorption and leads to the nonuniform broadening of resonance frequency of quantum transition in heterostructures. The inhomogeneity can lead to the smooth coordinate dependence ω_0 into the plane of quantum well or into the volume of superlattice, that leads to the broadening of lines in reflectivity and absorption spectra. Sharp lines of absorption and reflection are a quality indicator of the structure with quantum wells. The figure 1 shows this clearly. The change of reflectivity in the region of electron transitions has values around 10 - 50% for structures α and β, and 10 - 90% for structure γ. One can see from spectra, that QW heterosturctures $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ by type γ are more perfect for optical investigations.

Figure 2, A shows reflection spectra (R) of QW heterojunction $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ (γ type) in polarizations P,P and S,S (P ~ E||Z and S ~ E⊥Z, Z is grow direction of QW layers). Interference fringes in polarization P,P shift insignificantly in comparison with the case of polarization S,S. Figure 2, B shows reflection (R) and wavelength modulated reflection (ΔR/Δλ) spectra in pllarization P,S and differences of refractive indices (Δn^i and $\Delta n^{KK} = n^{PP} - n^{SS}$). The interference in interval 0.6 - 1.3 eV was observed in wavelength modulated reflection spectra in crossed polarizers (see Fig. 2, B). Narrow lines round more broad bands and are observed in these spectra. Interference fringes are more evident at energies E < 0.995 eV (λ_0) and less recognized for E > 0.995 eV. The energy 0.995 eV corresponds to the isotropic wavelength λ_0. The difference of refractive indices in the region of strong dispersion is received by using the next relationship:

$$\frac{d(n_P - n_S)l}{\lambda_l l} - \frac{d(n_P - n_S)l - 1}{\lambda_l l - 1} = \pm 1; \qquad (6)$$

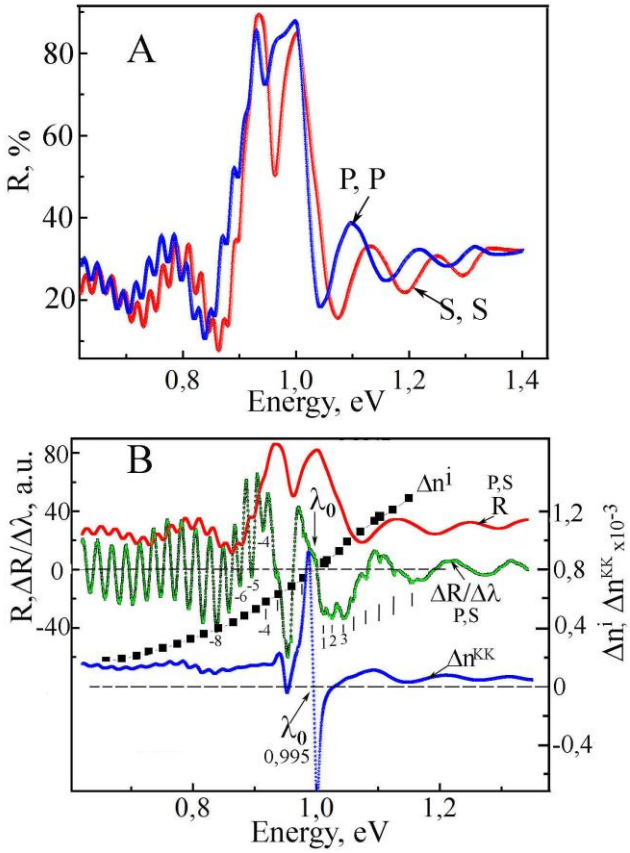

Fig. 2, A - Reflection spectra (R) of QW heterojunction $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ measured at the light wavelengths orientation in P,P and S,S polarizations. B - reflection (R) and wavelength modulated reflection (ΔR/Δλ) spectra measured in polarization P,S and differences of refractive indices calculated from interference fringes (Δn^i) and from reflection spectra using Kramers-Kronig relations ($\Delta n^{KK} = n^{PP} - n^{SS}$).

where birefringence at energies of maxima of the order of l is received in the units corresponded to l - 1, l is the number of maximum and the difference of refractive indices is given as an average of successive maxima (minima). The energy different between maxima (minima) in abovementioned equation is inversely proportional to the sample thickness. The value of refractive indices difference ($\Delta n^i = n_P - n_S$) marking numbers of fringes at short-wavelength (1, 2, 3 …) and long-wavelengths (-1, -2, -3 …) of the isotropic wavelength λ_0 = 1246 nm (0.995

eV) was calculated. The difference of refractive indices Δn^i is received from calculations of interference maxima and minima of $\Delta R/\Delta\lambda$ spectra and crosses the zero value of X-axis at wavelength λ_0 = 1246 нм (0.995 эВ). Assume that light waves propagate in waveguide layer of multiple QW refracting approximately 2000 times (equivalent with crystal thickness) the value of Δn of degree $0.2x10^{-3}$ have been received.

The valence band of $A^{III}B^V$ crystals is degenerated fourfold in the centre of Brillouin zone (q = 0). Bands of light and heavy holes have at q = 0. The uniaxial deformation leads to the shift of energy position of heavy and light holes. These bands are shifted with different degree at the uniaxial stress. The anisotropy of optical properties au in uniaxial crystals takes place in QW's heterostructures from cubic materials and grown along axes [100] or [111]. Formation of structure with QW's leads to the reducing of material symmetry in quantum wells. For quantum layers of $A^{III}B^V$ compounds grown along direction [001] the usual space group of zincblende Td is reduced to D_{2d}. This fact has an impact on optical properties - the linear birefringence and optical activity arise [1 - 3]. These effects are appreciable for the light propagating with wave-vector directed along quantum wells [7, 8].

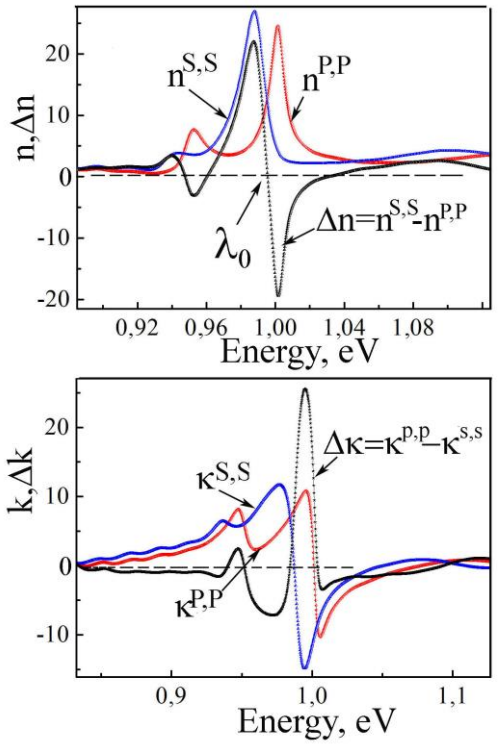

Fig. 3. Spectral dependences of refractive indices $n^{S,S}$, $n^{P,P}$ and they difference $\Delta n = n^{S,S} - n^{P,P}$ (top) and extinction coefficients $k^{S,S}$, $k^{P,P}$, $\Delta k = k^{P,P} - k^{S,S}$ (bottom) of QW structures.

Not only birefringence reveals in structures with QW as in uniaxial material of D_{2d} group, but also and an optical gyration. The nondiagonal component of the dielectric constant tensor is linearly depended on wave-vector q, for q parallel axes [100] or [010]. This effect does not reveal for q directed along [110] due to the existence of vertical plane of reflection (110) [1 - 3]. The existence of optical gyration leads to the optical activity, i.e. to different refractive indices for right- and left- circular polarized light. This can be observed then the birefringence is small. Thus the experimental separation of the optical gyration and birefringence is a reasonably difficult task. Because birefringence is much more then gyration by value. The existence of an isotropic point, i.e. the certain frequency for which linear birefringence vanishes, restores the isotropy for linear polarized light and, so provides a separation of both effects. The optical gyration is not usual observed away from the isotropic wavelength. Investigations of birefringence in QW structures $In_{0.68}Al_{0.1}Ga_{0.13}As/In_{0.42}Al_{0.22}Ga_{0.24}As$ grown on InP substrate were presented in this work. The experimental results demonstrate clearly the presence of interference spectra which change the fringes frequency at energy 0.995 eV (see Fig. 2). This is evidence of the existence of isotropic wavelength (λ_0) at this energy.

Calculations of reflection spectra with using of Kramers-Kronig relations were carried out for determination of optical constants dependences of QW $In_{0.68}Al_{0.1}Ga_{0.13}As/In_{0.42}Al_{0.22}Ga_{0.24}As$ structures. Figures 3 and 4 show spectral dependences of optical functions n, k, ε_1 and ε_2 at room temperature for energies of resonance transitions hh1-e1 and lh1-e1.

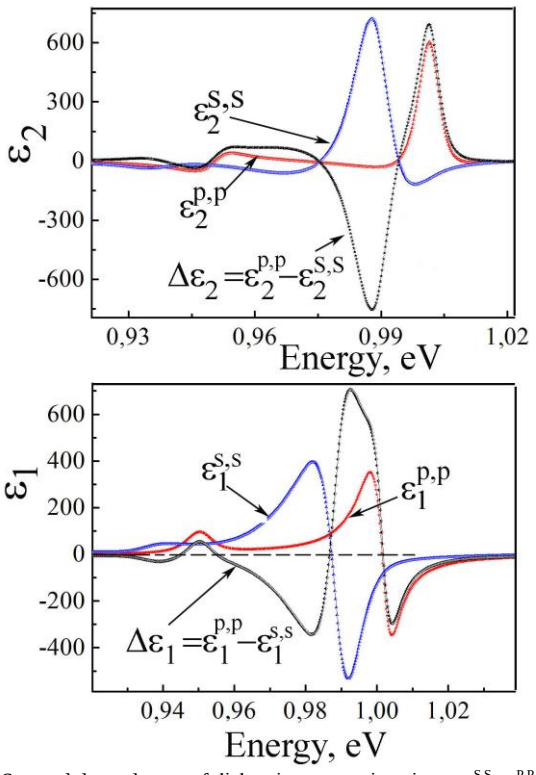

Fig. 4 Spectral dependences of dielectric constant imaginary $\varepsilon_2^{S,S}$, $\varepsilon_2^{P,P}$, $\Delta\varepsilon_2 = \varepsilon_2^{P,P} - \varepsilon_2^{S,S}$ (top) and real parts $\varepsilon_1^{S,S}$, $\varepsilon_1^{P,P}$, $\Delta\varepsilon_1 = \varepsilon_1^{P,P} - \varepsilon_1^{S,S}$ (bottom) of QW structures.

Spectral dependences of refractive index (n), extinction coefficient (k), real (ε_1) and imaginary (ε_2) parts of dielectric constant for polarized light waves have significant features in area of 0.9 - 1.1 eV. These features are caused by electron transitions hh1-e1 and lh1-e1 (Fig. 3 and Fig. 4). Functions ε_1 and ε_2 achieved values ≈ 400 and 600 at energies 0.98 and 1.0 eV for S,S and P,P polarizations, respectively (see Fig. 4). This

indicates that the largest absorption is observed at resonance of transition lh1-e1. The presence of electron transitions with different energy gaps becomes apparent in spectral dependences of optical functions. Due to the symmetry of energy bands, the light waves polarized along axis of layers grow Z excite only electron transitions hh1-e1, i.e. they correspond to the lowest energy gap. Light waves polarized perpendicular to axis Z associate with both energy gaps hh1-e1 and lh1-e1, sc. they excite electron transitions from both levels - heavy and light holes. Spectral dependences of optical functions differences $\Delta n = n^{S,S} - n^{P,P}$, $\Delta k = k^{P,P} - k^{S,S}$, $\Delta\varepsilon_2 = \varepsilon_2^{P,P} - \varepsilon_2^{S,S}$ и $\Delta\varepsilon_1 = \varepsilon_1^{P,P} - \varepsilon_1^{S,S}$ of QW structures $In_{0.68}Al_{0.1}Ga_{0.13}As/\ In_{0.42}Al_{0.22}Ga_{0.24}As$ cross the zero axis at energy of isotropic wavelength (0.995 eV).

As mentioned above in QW layers subbands of light and heavy holes are split in Brillouin zone center. Effective masses of holes and electrons are changed in QW's. These factors influence on the value of background dielectric constant nearby an electron (exciton) resonance. A presence of two nearest resonances in quantum well is taken into account with help the next dielectric function of quantum well material:

$$\varepsilon(\omega,q)=\varepsilon_b\left(1+\sum_{j=1}^{2}\frac{2\omega_{LT}^{(j)}\omega_0^{(l)}}{\left(\omega_0^{(j)}\right)^2-\omega^2+\frac{hq^2\omega_0^{(l)}}{M^{(j)}}-i\omega\Gamma^{(0)}}\right), \quad (7)$$

where ω is a frequency, q is a magnitude of wave-vector, ε_b is a background dielectric constant. The index of resonance j accepts a value j = 1 for transitions from bands of heavy holes (hh) and j = 2 for electronic transitions from bands of light holes (lh). As a rule only one resonance mode is considered [2 - 7]. For resonance of j[th] type: $\omega_0^{(j)}$ is resonance frequency, $\omega_{LT}^{(j)}$ is longitudinal-transversal splitting frequency, $\Gamma^{(j)}$ is damping parameter, $M^{(j)}$ is a translation mass of transition in a quantum well. In the case of two energetically nearest electron transitions like resonances connected with light and heavy holes the correlation between $\omega_{LT}^{(j)}$ and splitting of light and heavy holes bands is very important. The longitudinal-transversal splitting $\omega_{LT}^{(j)}$ could be more or less then the splitting of heavy and light holes bands. All this put some requirements for analysis of reflection and absorption optical spectra of light and heavy holes resonances.

In real structure with QW the brier layers have a finite thickness. One borders with vacuum and another with thick substrate. The fundamental absorption edge in substrate lies below then a resonance frequency of transitions in quantum well and refractive indices of substrate and barriers materials are very similar. Thus the interior barrier can be considered as half-infinfte thickness and a refractive index from all structure is written as [9 - 11]:

$$r=r_{01}+\frac{t_{01}t_{10}e^{2i\varphi}}{1-r_{10}r_{QW}e^{2i\varphi}}r_{QW}; \quad (8)$$

$$\varphi=n_b\frac{\omega}{c}\left(b+\frac{a}{2}\right); \quad (9)$$

Where r_{jl} and t_{jl} are amplitude coefficients of reflection and transmission on the border between mediums j and 1 (j, l = 0 in vacuum and 1 in barierr) which are calculated according Fresnel formula (11); r_{QW} is a reflection coefficient from quantum well determined by (10); φ is a phase shift which received light after passing a distance, this distance added from the thickness of outer barrier b and a half of well thickness a; c is light velocity in vacuum.

$$r(\omega)=\frac{i\Gamma_0}{\tilde{\omega}_0-\omega-i(\Gamma+\Gamma_0)}; \quad (10)$$

where $\tilde{\omega}_0$ is a renormalized resonance frequency of quantum junction; $\tau_0=(2\Gamma_0)^{-1}$ is a radiative lifetime; Γ is a nonradiative damping of quantum junction.

$$r_{10}=-r_{01}=\frac{n_b-1}{n_b+1};t_{01}t_{10}=\frac{4n_b}{(n_b+1)^2}. \quad (11)$$

In the last equation n_b is a refractive coefficient of outer barrier. Reflection coefficient for QW structure is estimated by formula:

$$R(\omega)=|r(\omega)|^2. \quad (12)$$

For several periodical quantum well r_{QW} in (8) is changed on r_N which is calculated by next expression:

$$r_N=\frac{iN\Gamma_0}{\tilde{\omega}_0-\omega-i(\Gamma+N\Gamma_0)}. \quad (13)$$

In expression (10) the radiative damping Γ_0 is replaced on $N\Gamma_0$, where N is number of wells (number of quantum layers). The formula for phase shift calculating has a look:

$$\varphi=\frac{\omega}{c}n_aNd, \quad (14)$$

where $n_a \approx n_b$ is a refractive index of quantum well material; N is number of quantum wells; d is distance between neighboring quantum wells.

Fig. 5 shows the experimentally (exp.) measured at 300 K reflection spectra and calculated (cal.) by dispersion equations contours for excitonic resonances [12]. Two intense maxima due to transitions hh1-e1 and lh1-e1 are singled out in the experimental reflection spectra of QW structures $In_{0.68}Al_{0.1}Ga_{0.13}As/\ In_{0.42}Al_{0.22}Ga_{0.24}As$. These features are very closed energetically and therefore, the calculation of reflectance spectra contours is made using two-oscillator model.

After fitting experimentally measured reflection spectra using two-oscillator model we have at 300 K in S,S polarization for an oscillator with a resonant frequency ω_{01} = 0.929 eV - $\Delta\omega_{LT}$ = 26 meV, Γ = 5.5 meV and ε_b = 5.7, and for an oscillator with a resonant frequency ω_{02} = 0.983 eV - $\Delta\omega_{LT}$ = 45 meV, Γ = 4.5 meV and ε_b = 5.7. The change of the effective masses ($M = m_e + m_h$) values have a little effect on contours of calculated curves, but nevertheless they are

different for both oscillators. For the oscillator ω_{01} mass M = $4.3m_0$ and for oscillator ω_{02} M = $8.6m_0$. In P,P polarization we have the resonance frequency of the long-wavelength oscillator ω_{01} = 0.929 eV, $\Delta\omega_{LT}$ = 12 meV, Γ = 6.0 meV and ε_b = 5.9. The resonant frequency of the short-wavelength oscillator ω_{02} = 0.991 eV, $\Delta\omega_{LT}$ = 23 meV, Γ = 6.0 meV and ε_b = 5.9. For the first oscillator (ω_{01}) mass is M = $4.8m_0$ and for oscillator (ω_{02}) the mass equals M = $9.5m_0$. One can see that effective masses M in both polarizations (S,S and P,P) are different. The translational mass (M) of heavy holes transitions (hh1-e1) is twice less than the mass obtained for light hole band transitions (lh1-e1). Since the translational mass (M) is the sum of electrons and holes effective masses and electrons are found only in one zone e1, therefore the effective mass of the heavy holes is half of the effective mass of the light holes. This method was applied for calculation of exciton-polaritons spectra of bulk crystals [12], and used by us for an evaluation of effective masses and determination of the background dielectric constant (ε_b) in and around of electronic transitions in QW.

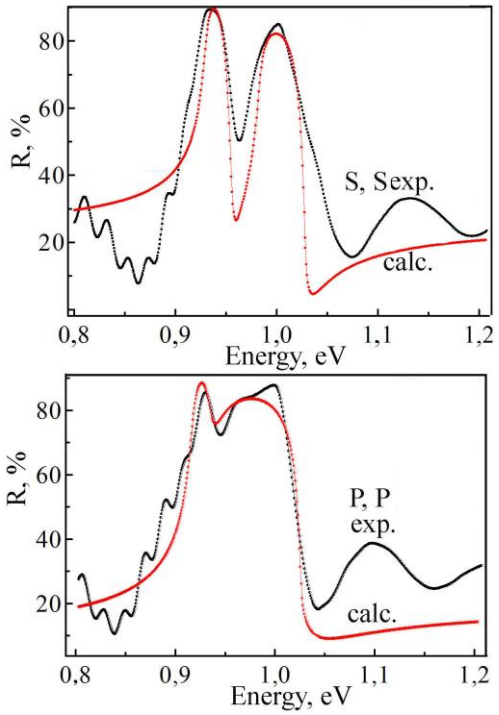

Fig. 5 Contours of reflection spectra of QW structure measured experimentally (exp.) at temperature 300 K for polarizations S,S (top) and P,P (bottom) and calculated using dispersion equations (calc.) in the framework of model for excitons.

Fig. 6 shows the experimental (exp.) and calculated (calc.) reflection spectra in the framework of model for quantum transitions [9 - 11]. The background dielectric constant (ε_b) values in resonances hh1-e1 and lh1-e1 of QW structures $In_{0.68}Al_{0.1}Ga_{0.13}As/In_{0.42}Al_{0.22}Ga_{0.24}As$ in S,S and P,P polarizations are equal to 11.9 and 12.0, respectively. In S,S polarization for the quantum transition hh1-e1 coefficient Γ = 2.0 and Γ_0 = 6.1 and for lh1-e1 transitions Γ = 5.2 and Γ_0 = 15.0 (Γ characterizes the nonradiative damping, Γ_0 - the radiative lifetime of quantum transition). In P,P polarized light waves for quantum transitions hh1-e1 coefficient Γ = 4.2 and Γ_0 = 12.1 and for lh1-e1 transitions Γ = 4.0 and Γ_0 = 12.0. The radiative lifetime (τ_0) is less than nonradiative lifetime (τ) of charge carriers of transitions hh1-e1 and lh1-e1 in both polarizations. Simultaneously, it should be noted that P,P polarization τ_0 abd τ for transitions hh1-e1 and lh1-e1 coincide. In the case of S,S polarization, τ_0 and τ for transitions hh1-e1 and lh1-e1 differ by 2.5 times. Light holes effective masses are greater twice than heavy holes masses. It can be supposed that light holes in the S,S polarization responsible largely for τ_0 and τ, than in the case of P,P polarization.

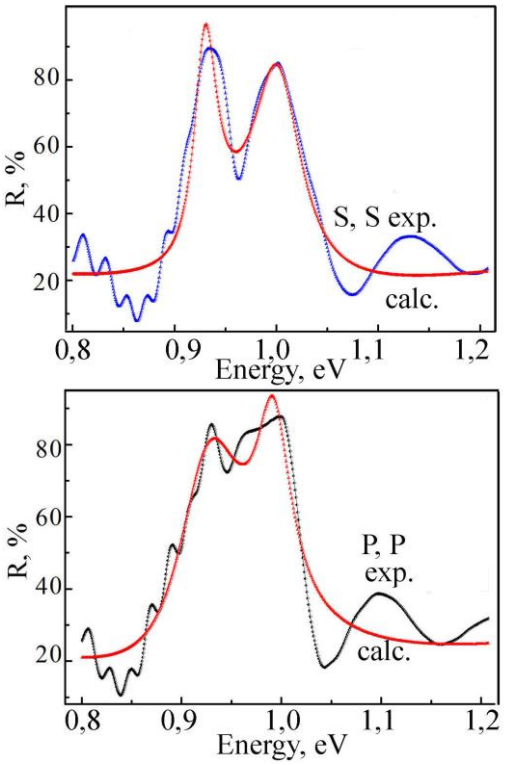

Fig. 6 Contours of reflection spectra of QW structure measured experimentally (exp.) at temperature 300 K for polarizations S,S (top) and P,P (bottom)and calculated using dispersion equations (calc.) in the framework of model for quantum transitions.

IV. Conclusions

Studies of reflection spectra at light waves orientation in polarizations P,P and S,S and wavelength modulated reflection spectra in crossed polarizations P,S of QW heterojunction $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$ indicate the presence of light waves birefringence in QW layers. Optical functions were calculated from polarized reflection spectra by Kramers-Kronig analysis. The anisotropy of optical functions spectral dependences like: refractive index ($n^{S,S}$, $n^{P,P}$, $\Delta n = n^{S,S} - n^{P,P}$), extinction coefficient ($k^{S,S}$, $k^{P,P}$, $\Delta k = k^{P,P} - k^{S,S}$), real and imaginary parts of dielectric constant ($\varepsilon_1^{S,S}$, $\varepsilon_1^{P,P}$, $\Delta\varepsilon_1 = \varepsilon_1^{P,P} - \varepsilon_1^{S,S}$, $\varepsilon_2^{S,S}$, $\varepsilon_2^{P,P}$, $\Delta\varepsilon_2 = \varepsilon_2^{P,P} - \varepsilon_2^{S,S}$) of QW structure were revealed. The interference of polarized light waves in the QW layers indicates the presence of isotropic wavelength λ_0 - 1.246 μm. Interference fringes change their periodicity and refractive

indices differences (Δn^i, $\Delta n^{KK} = n^{PP} - n^{SS}$) cross the zero axis at isotropic wavelength energy (0.995 eV).

The difference in the effective masses (M) in polarizations S,S and P,P was revealed and evaluated from calculations of reflection spectra contours provided by transitions hh1-e1 and lh1-e1 of QW structures $In_{0.68}Al_{0.1}Ga_{0.13}As/ In_{0.42}Al_{0.22}Ga_{0.24}As$. The translational mass (M) of transitions hh1-e1 is twice less than the mass for transitions lh1-e1, that indicates the difference between the effective masses of heavy (m_{hh}) and light (m_{lh}) holes.

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