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HARMONIC ANALYSIS OF MAGNETOMOTIVE FORCE IN THE AIR GAP OF SIX-PHASE ASYNCHRONOUS MACHINES

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Abstract. Asynchronous motors with six and more phases (usually n-three-phase) are increasingly accepted in electric drive systems of road transport, marine and even aeronautical transport. Multiphase machines are credited with multiple advantages such as: less torque ripple, better power distribution per phase, higher efficiency and fault tolerance capability compared to traditional three-phase machines. There are many publications, but few of them are dedicated to the in-depth study of the multiphase motor (m>3) and especially of the magnetic field in its air gap. The purpose of the present paper: the development of the mathematical model of the magnetomotive force in the air gap of the six-phase asynchronous machines, which determines the fundamental properties of the machine, the structure of the magnetic field and the torque developed, the efficiency, the error tolerance. The research is based on analytical studies and the results of suitable experimental measurements. The paper presents the mathematical model for the harmonic analysis of the magnetomotive force in the air gap of six-phase machines with possible variants of windings on the stator. Based on this study, it was demonstrated that only n-three-phase machines with asymmetrically arranged stator windings have the ability to attenuate higher harmonics with a major negative effect on the developed torque, the applicability and efficiency of traditional tools for reducing higher harmonics in the case of six-phase machines was confirmed. The results of measurements carried out on samples of six-phase machines confirm the findings and conclusions formulated in the paper.

Keywords: *Induction machine, six-phase machine, air-gap magnetomotive force, harmonic analysis, asynchronous torques*

Rezumat. Motoarele asincrone cu șase și mai multe faze (de regulă, n-trifazate) tot mai des sunt acceptate în cadrul sistemelor de acționare electrică a mijloacelor de transport rutier, naval și chiar aeronautic. Mașinilor multifazate li se atribuie multiple avantaje, cum ar fi: ondularea mai mică a cuplului, distribuția mai bună a puterii pe fază, eficiența mai mare și capacitatea de toleranță la erori în comparație cu mașinile trifazate tradiționale. Sunt multe publicații, dar puține din acestea sunt consacrate studiului aprofundat al motorului multifazat (m>3) și în deosebi a câmpului magnetic în întrefierul acestuia. Scopul prezentei lucrări: elaborarea modelului matematic al forței magnetomotoare în întrefierul mașinilor asincrone hexafazate, care determină proprietățile fundamentale ale mașinii, structura câmpului magnetic și a cuplului dezvoltat, eficiența, toleranța la erori. Cercetarea se bazează pe studii analitice și pe rezultate ale măsurătorilor experimentale adecvate. În lucrare este prezentat modelul matematic pentru analiza armonică a forței magnetomotoare în întrefierul mașinilor hexafazate cu variante posibile de înfășurări pe stator. În baza acestui studiu, s-a demonstrat că doar mașinile n-trifazate cu înfășurări statorice dispuse asimetric dispun de capacitatea de atenuare a armonicilor superioare cu efect negativ major asupra cuplului dezvoltat, s-a confirmat aplicabilitatea și eficiența instrumentelor tradiționale de reducere a armonicilor superioare în cazul mașinilor hexafazate. Rezultatele măsurătorilor efectuate pe mostre de mașini hexafazate confirmă constatările și concluziile formulate în lucrare.

Cuvinte cheie: *Mașina de inducție, mașina hexafazată, forța magnetomotoare în întrefier, analiza armonică, cupluri asincrone.*

1. Introduction

Multiphase asynchronous motors (m>3) are becoming more and more popular for the electric drive of road, marine and even aeronautical vehicles due to their advantages such as less torque ripple, better power distribution per phase, efficiency higher and fault tolerance compared to three-phase ones [1-3]. An increased preference is observed in the application of n-three-phase machines which, in parallel with the simplicity of the construction and the simplicity of the manufacturing technologies, which do not differ from those applied to the production of conventional three-phase machines, add the operational advantages named above [4-7]. They are distinguished by the arrangement on the stator of 2, 3 or more identical three-phase sets of windings offset from each other at a predetermined angle (Figures 2b and 2c).

The study of the magnetic field in the air gap of electric motors has been an actual problem throughout the historical course of their development. In the gap, the electromagnetic and electromechanical transformations occur that determine the energy parameters but also the quality of the machine's operating characteristics [9-12]. Multiphase machines are no exception in this field either - especially n-three-phase ones. Results of the fundamental analytical research of the three-phase induction machine can be found in the works of R. Rihter [13], V. Radin [14], in the treatises of professors M. Covrig [10], A. Simion [15], I. Boldea [16], S. Deaconu [17], I. Kopilov [18], I. Boldea and Nasar S.A. [19]. And in recent publications the problem of the field and parasitic couples in the healthy regime and in the operation with errors in the power system is widely discussed [11, 12, 20].

In several works, the six-phase machines, which will be discussed in this paper, are attributed important advantages / performances regarding the reduction of the parasitic harmonics of the induction in the air gap, which ensures a better uniformity of the developed torque but also of reducing losses of energy in the machine compared to classic three-phase machines. However, it seems that none of these works specifies to which type of six-phase machines some or others of these advantages refer. The purpose of this paper is to determine the most suitable variants of winding topographies/architectures to achieve optimal levels of efficiency and reliability of electrical drive systems with n-three-phase induction machines.

Research objectives: a) performing a deep study of the harmonic structure of the magnetomotive force (MMF) in the air gap of the n-three-phase induction machine, b) defining the link between the harmonic structure, the type and parameters of the windings used, c) tracing the methods of eliminating or reducing the negative effect of the higher harmonics of the MMF in six-phase machines (with various types of windings). The work is oriented towards an analytical study accompanied by arguments based on the results of the measurements performed on suitable experimental samples.

2. The magnetomotive force in the air gap of the induction machine with the multiphase stator winding

One of the main concerns in the design of electric machines is centered on the correct determination of the topology and constructive parameters of the windings, aiming to minimize the content of existing harmonics in the waveform of the magnetomotive force of the air gap.

The fundamental harmonic of the MMF results in a rotating magnetic flux and finally creates a mechanical torque that produces the useful work [10]. At the same time, the presence of higher spatial harmonics in the rotating wave of the MMF results in the formation of parasitic asynchronous torques, accompanied by power losses and the reduction of the energy efficiency of the machine. Another group of harmonics, of order multiple to 3, in the case of n-three-phase machines, forms pulsating waves accompanied by additional losses in the magnetic core, winding conductors and other metal parts [13-17]. In what follows, analytically, the spectrum of spatial harmonics of MMF in the gap of n-three-phase type machines will be determined. The final goal of the study being - highlighting the types of windings advantageous from the point of view of the spectrum of higher harmonics that produce parasitic asynchronous torques, with a significant negative impact on the machine's operating characteristics.

2.1 Magnetomotive force created by a single-phase winding

A single-phase winding (one coil), concentrated, with the number of turns w (Figure 1a) was examined. Each side of the coil occupies a slot of the ferromagnetic armatures, q=1, and the distance between the sides corresponds to a polar step y=τ. The number of pairs of poles, formed by the magnetic field is p=1. If the current that runs through the coil is sinusoidal and is described with the relation (1):

$$i(t) = I_m \sin(\omega t), \tag{1}$$

where: I_m - is the amplitude of the current, $\omega=2\pi f$ - the pulsation, and f - represents the frequency of variation of the current intensity, then, the magnetomotive force along a polar step is represented by the relation (2):

$$F(X) = \begin{cases} +\frac{F_0}{2}, \text{ if } \quad 0 < X < \pi \\ -\frac{F_0}{2}, \text{ if } \quad 0 > X > -\pi \end{cases} \tag{2}$$

where:

$$F_0 = I_m\, w\, k_w \ \sin(\omega t) = \sqrt{2}\, I\, w\, k_w \ \sin(\omega t) = 2F_m \sin(\omega t) \tag{3}$$

$X = \pi \cdot x/\tau$ - represents the distance along the curvature of the air gap, expressed in electrical degrees, x - represents the coordinate in metric expression, k_w - winding factor, I - the effective value of the current intensity, w- the number of turns of the winding.

For a given value of the current, for example, when ($\omega t = \pi/2$ and $\sin(\omega t) = 1$), the MMF distribution along the air gap is shown graphically with a step diagram as in figure 1a, this curve being a symmetric periodic function, the Fourier expansion does not contain terms of even order and is represented by the relation:

$$F(x) = \frac{\sqrt{2}}{2}\, Iw\, k_w \sin(\omega t)\, [\, a_1 \cos\frac{\pi x}{\tau} + a_3 \cos\left(3\, \frac{\pi x}{\tau}\right) + \cdots + a_\nu \cos\left(\nu\, \frac{\pi x}{\tau}\right)] \tag{4}$$

where $a_1, a_2, \ldots, a_\nu$ are the Fourier coefficients, which are defined as follows:

$$a_\nu = \frac{1}{\pi}\int_{-\pi}^{+\pi} \cos(\nu X)\, dX = \frac{4}{\pi}\int_0^{\pi/2} \cos(\nu X)\, dX = \frac{4}{\pi\nu}\sin\left(\frac{\nu\pi}{2}\right) \tag{5}$$

Inserting into (4) the coefficients a_ν, defined with relation (5), it results:

$$F(x) = \frac{2\sqrt{2}}{\pi}\, Iw \sin(\omega t)\, \sum_1^n [\frac{k_{w.\nu}}{\nu} \cos\left(\nu\, \frac{\pi x}{\tau}\right)] \tag{6}$$

Thus, the MMF in the airs gap is represented by the fundamental harmonic (curve 1 in Figure 1b) and a lot of higher spatial harmonics (e.g. curves 3 and 5 in Figure 1b).

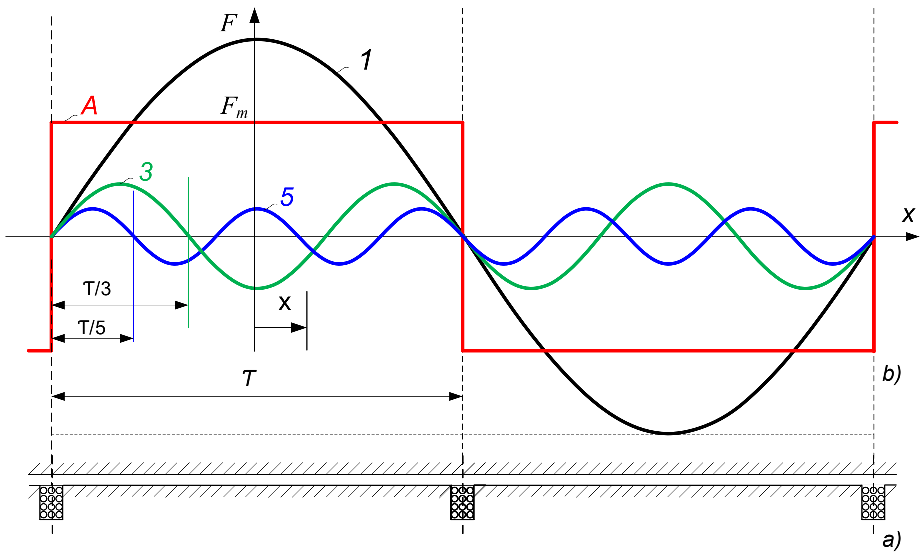


Figure 1. a) Air gap geometry (with the winding placed in the notches); b) MMF distribution along the air gap of an electric machine with a single-phase concentrated winding (curve A), and the component harmonics: fundamental (curve 1) and higher ones (curves 3 and 5).

From (6) it follows that a harmonic, with order number ν, is described by the relation:

$$F_\nu = F_{m.\nu} \sin(\omega t) \cos\left(\nu\, \frac{\pi x}{\tau}\right) = \frac{2\sqrt{2}}{\nu\, \pi} I\, w\, k_{w.\nu} \sin(\omega t) \cos\left(\nu\, \frac{\pi x}{\tau}\right) \tag{7}$$

The amplitude $F_{m.\nu}$ of the harmonic of order ν can be related to the amplitude of the fundamental $F_{m.1}$ and then:

$$F_\nu = \frac{F_{m.1}}{\nu}\frac{k_{w.\nu}}{k_{w.1}} \sin(\omega t) \cos\left(\nu\, \frac{\pi x}{\tau}\right) = F_{m.1}\frac{k'_{w.\nu}}{\nu} \sin(\omega t) \cos\left(\pi x\, \frac{\nu}{\tau}\right) \tag{8}$$

Here it was noted: $k_{w.1}$, $k_{w.\nu}$ – the winding factor, calculated for the fundamental and harmonic respectively ν, $k'_{w.\nu}$ the winding factor for the harmonic ν reported, $F_{m.1} = \frac{2\sqrt{2}}{\nu\, \pi} Iw$ - amplitude of the fundamental.

From relation (8) it is observed that the amplitude $F_{m.\nu}$ and the wavelength τ_ν decrease proportionally to the order number ν of the harmonic. A higher harmonic varies in time with the frequency f_1 but creates in the gap $(\nu \cdot p)$ pole pairs.

The magnetomotive force of a three-phase machine is defined by the MMF sum of three phase windings, described based on relations (6) and (8). But the MMF of 6-, 9- or 12-

phase machines (in general - of an n-three-phase machine) can be defined by the sum of the MMF of n sets of three-phase windings offset from each other in space at a prescribed angle θ. Figures 2a, 2b and 2c show the orientation diagrams of the axes of the phase windings of 3- and 6-phase induction machines. Of course, the phase shift λ of the phasors of the currents passing through the phases of the three-phase sets of windings will have the value equal to the gap θ, expressed in electrical degrees (Figures 2d, 2e and 2f). Figure 2 shows the generalized diagram for a machine with 2p=2.

Starting from these considerations, in the following the relations describing the upper harmonics (of order ν) of the MMF are defined for each phase separately, then of a three-phase set, later the variants practiced by six-phase composite windings are examined.

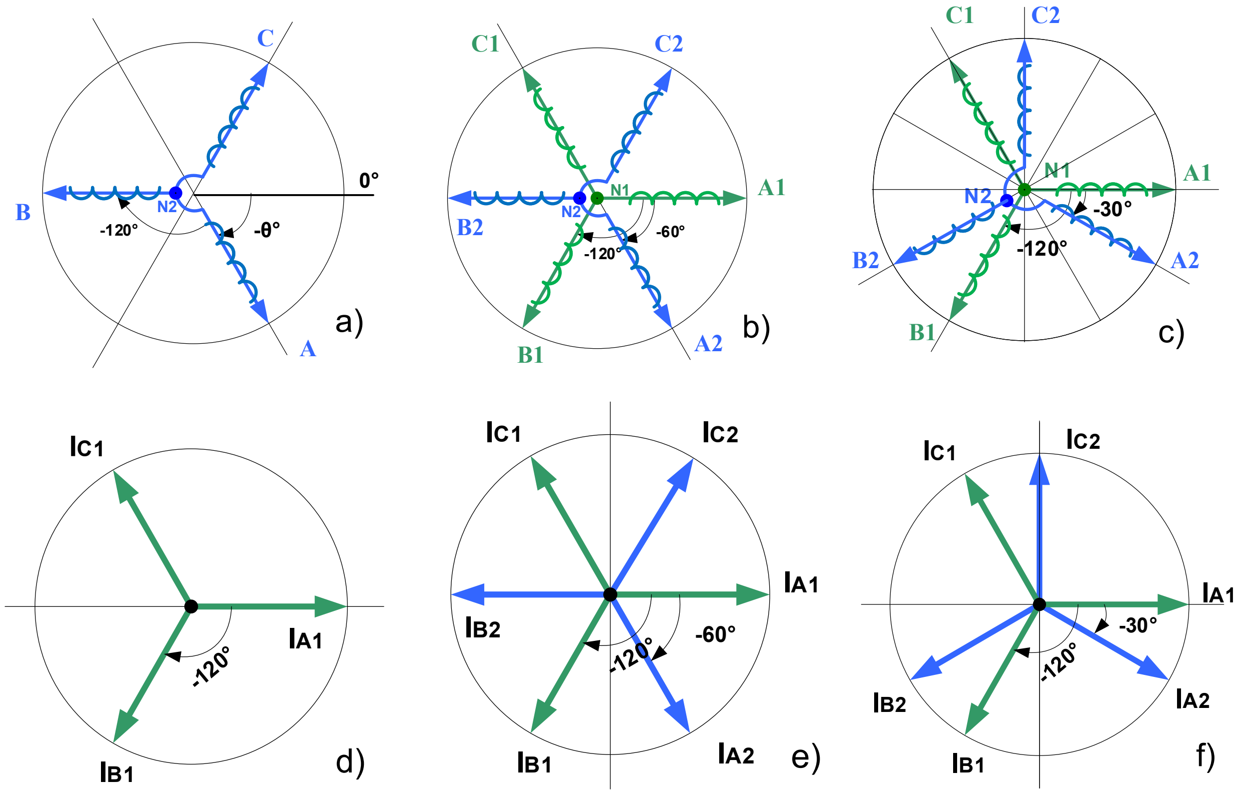


Figure 2. Stator winding type: a) three-phase; b) symmetrical hexaphase; c) asymmetrical hexaphase; d)-f) the phasors of the phase currents in the respective windings.

2.2 MMF in the air gap of the three-phase induction machine

In the case of a three-phase set of windings (Figure 2a), the phases being marked, for example, with A, B and C, are oriented under 120 electrical degrees to each other, and the three-phase set as a whole has a spatial offset of θ electrical degrees, in relation to the axis defined as reference (θ=0). Under these conditions, the relations for the ν - order harmonic of the phase MMF are presented as follows:

$$F_{A\nu} = F_{m1}\frac{k'_{w.\nu}}{\nu}\sin(\omega t-\theta)\cos\left(\nu\frac{\pi x}{\tau}-\nu\theta\right) \tag{9}$$

$$F_{B\nu} = F_{m1}\frac{k'_{w.\nu}}{\nu}\sin(\omega t-\frac{2\pi}{3}-\theta)\cos\left(\nu\frac{\pi x}{\tau}-\nu\left(\frac{2\pi}{3}+\theta\right)\right) \tag{10}$$

$$F_{C\nu} = F_{m1}\frac{k'_{w.\nu}}{\nu}\sin\left(\omega t-\frac{4\pi}{3}-\theta\right)\cos\left(\nu\frac{\pi x}{\tau}-\nu\left(\frac{4\pi}{3}+\theta\right)\right) \tag{11}$$

Under these conditions, the rotating components of the ν-order harmonic of the MMF ($F'_{A\nu}$-direct and $F''_{A\nu}$-inverse) of the phase windings can be presented with relations (12)-(14):

$$F_{A\nu} = F'_{A\nu} + F''_{A\nu} = \frac{F_{m1}}{2}\frac{k'_{w.\nu}}{\nu}[\sin(\varphi_{1\nu} + (\nu-1)\,\theta) + \sin(\varphi_{2\nu} - (\nu+1)\,\theta)] \quad (12)$$

$$F_{B\nu} = F'_{B\nu} + F''_{B\nu} = \frac{F_{m1}}{2}\frac{k'_{w.\nu}}{\nu}[\sin\left(\varphi_1 + (\nu-1)\,(\frac{2\pi}{3} + \theta\right) + \sin\left(\varphi_2 - (\nu+1)\left(\frac{2\pi}{3} - \theta\right)\right)] \quad (13)$$

$$F_{C\nu} = F'_{C\nu} + F''_{C\nu} = \frac{F_{m1}}{2}\frac{k'_{w.\nu}}{\nu}[\sin\left(\varphi_1 + (\nu-1)\,(\frac{4\pi}{3} + \theta\right) + \sin\left(\varphi_2 - (\nu+1)\left(\frac{4\pi}{3} - \theta\right)\right)] \quad (14)$$

where: $\varphi_{1\nu} = \omega t - \nu\frac{\pi x}{\tau}$ - is the phase angle of the direct spin wave, and $\varphi_{2\nu} = \omega t + \nu\frac{\pi x}{\tau}$ - is the phase angle of the inverse rotating wave of the harmonic ν.

Applying the decomposition $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$ and simplifying, based on relations (12-14), the following convenient relations are obtained for a tabular calculation of the components of the rotating wave of the MMF:

$$F'_{\nu} = \frac{F_{m1}}{2}\frac{k'_{w.\nu}}{\nu}[k_{1\nu}\sin(\varphi_{1\nu}) + k_{2\nu}\cos(\varphi_{1\nu})\,] \quad (15)$$

$$F''_{\nu} = \frac{F_{m1}}{2}\frac{k'_{w.\nu}}{\nu}[k_{3\nu}\sin(\varphi_{2\nu}) - k_{4\nu}\cos(\varphi_{2\nu})\,] \quad (16)$$

where $k_{1\nu}, k_{2\nu}, k_{3\nu}, k_{4\nu}$ - called harmonic coefficients, are defined relations (17):

$$\begin{cases} k_{1\nu} = \cos\big((\nu-1)\theta\big) + \cos\left((\nu-1)\left(\frac{2\pi}{3}+\theta\right)\right) + \cos\left((\nu-1)\left(\frac{4\pi}{3}+\theta\right)\right) \\ k_{2\nu} = \sin\big((\nu-1)\theta\big) + \sin\left((\nu-1)\left(\frac{2\pi}{3}+\theta\right)\right) + \sin\left((\nu-1)\left(\frac{4\pi}{3}+\theta\right)\right) = 0 \\ k_{3\nu} = \cos\big((\nu+1)\theta\big) + \cos\left((\nu+1)\left(\frac{2\pi}{3}+\theta\right)\right) + \cos\left((\nu+1)\left(\frac{4\pi}{3}+\theta\right)\right) \\ k_{4\nu} = \sin\big((\nu+1)\theta\big) + \sin\left((\nu+1)\left(\frac{2\pi}{3}+\theta\right)\right) + \sin\left((\nu+1)\left(\frac{4\pi}{3}+\theta\right)\right) = 0 \end{cases} \quad (17)$$

Table 1 shows the results of the calculations performed for a three-phase winding in star connection for four scenarios of offset θ of phase A in relation to the reference axis.

Following the analysis of the results, included in this table, it is found:

a) the harmonic coefficients in all the examined scenarios obtain the value 0 or ±m, where m is the number of phases of the winding, and the minus sign in front notifies a phase shift of 180 electrical degrees of the respective rotating wave;

b) the even harmonic coefficients ($k_{2\nu}, k_{4\nu}$), for all examined values of the gap θ, have zero value, regardless of the harmonic order. Therefore, the spin waves (forward and reverse) in all the scenarios examined are sinusoidal (see relations (15) and (16));

c) MMF waves of order $(2m \cdot k + 1)$ have the direction of direct rotation, and those of order $(2m \cdot k - 1)$ rotate in the opposite direction (they are of opposite sequence). Here k – any integer.

Table 1

Calculated harmonic coefficients for a three-phase winding (m=3) with different values θ of the offset angle θ with respect to the reference axis

ν	The offset θ of the three-phase set of windings with respect to the reference axis (θ=0):															
	Scenario 1: $\theta = 0°$				Scenario 2: $\theta = 30°$				Scenario 3: $\theta = 60°$				Scenario 4: θ=90°			
	k1.1	k2.1	k3.1	k4.1	k1.2	k2.2	k3.2	k4.2	k1.3	k2.3	k3.3	K4.3	k1.4	k2.4	k3.4	k4.4
1	3	0	0	0	3	0	0	0	3	0	0	0	3	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	3	0	0	0	-3	0	0	0	3	0	0	0	-3	0

Continuation Table 1

7	3	0	0	0	-3	0	0	0	3	0	0	0	-3	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	3	0	0	0	3	0	0	0	3	0	0	0	3	0
13	3	0	0	0	3	0	0	0	3	0	0	0	3	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	3	0	0	0	-3	0	0	0	3	0	0	0	-3	0
19	3	0	0	0	-3	0	0	0	3	0	0	0	-3	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	3	0	0	0	3	0	0	0	3	0	0	0	3	0
25	3	0	0	0	3	0	0	0	3	0	0	0	3	0	0	0

Note: ν - harmonic number (order harmonic); k - harmonic coefficients.

2.3 MMF of the air gap of the six-phase induction machine

For a six-phase machine (Figures 2b and 2c) the harmonic coefficients are defined by summing the coefficients calculated for the three-phase sets that make up the machine winding. Table 2 shows the results of the calculations for four possible scenarios for the execution of the six-phase winding: a) symmetrical double-three-phase type with the gap between sets θ=0°, b) symmetrical type with θ=60°, c) and d) asymmetrical type with the gap between sets of 30 or 90 electrical degrees. Only the values of the synthetic coefficients K1 and K3 are presented in the table, given the fact that the coefficients K2 and K4 in all the examined scenarios have zero value.

Table 2

Calculated harmonic coefficients for 4 scenarios for making six-phase windings

m=6	SIX-PHASE							
	Double 3-phase		**Symmetric, θ= 60°**		**Asymmetric, θ=30°**		**Asymmetric, θ= 90°**	
ν	K1= 2 k11	K3=2 k31	K1= k11+k13	K3= k31+k33	K1= k11+k12	K3= k31+k32	K1= k11+k14	K3= k31+k34
1	6	0	6	0	6	0	6	0
3	0	0	0	0	0	0	0	0
5	0	6	0	6	0	0	0	0
7	6	0	6	0	0	0	0	0
9	0	0	0	0	0	0	0	0
11	0	6	0	6	0	6	0	6
13	6	0	6	0	6	0	6	0
15	0	0	0	0	0	0	0	0
17	0	6	0	6	0	0	0	0
19	6	0	6	0	0	0	0	0
21	0	0	0	0	0	0	0	0
23	0	6	0	6	0	6	0	6
25	6	0	6	0	6	0	6	0

Note: ν - harmonic number (order harmonic); k - harmonic coefficients; θ - phase shift.

This calculation algorithm can also be applied to other machines from the n-three-phase group: with 9, 12, 15, etc. phases. For example, the winding of the 12-phase machine can be synthesized from 4 three-phase sets offset by 30 electrical degrees. The calculation results demonstrate that the harmonic composition of the MMF will be similar to that of the asymmetrical six-phase machine.

Following the analysis of the obtained results (Table 2), the following practical conclusions can be drawn regarding the harmonic structure of the MMF and respectively of the magnetic field in the gap of an n-three-phase electric machine (it is about the spatial harmonics because the phase currents are considered sinusoidal):

a) the magnetomotive force created in the gap by the phase windings of the multiphase induction machine contains only odd harmonics whose order is determined by the relation: $\nu=2k\pm1$, where k=0, 1, 2, 3, etc.;

b) in the case of n- three-phase machines (with 3, 6, 9, 12 etc. phases), the third order harmonics in the phases of each three-phase set of windings have a gap of 0 between them, and do not form rotating magnetic fields, they only form pulsating fields. This phenomenon refers to all harmonic multiples of 3 (e.g. 3, 9, 15, 21, 27, 33...).

c) increasing the number of three-phase sets within the multiphase stator winding leads to the annihilation / disappearance of higher harmonics only when they are arranged asymmetrically. For example, from the harmonic spectrum of the MMF of the asymmetric hexaphase machine, the harmonics are missing: 5, 7, 17, 19, 29, 31 ... (k m±1), where k=1, 3, 5 ...

d) thus, it is found that in an asymmetric hexaphase machine the first of the higher harmonics of the MMF, which forms a rotating magnetic field, is the eleventh order. In the case of the machine with m=12, the first in this sense is the 23rd harmonic and has a relative amplitude value of at most 4.3%;

e) the hexaphase machine with the double-three-phase winding or the symmetrical n-three-phase type behaves like a simple three-phase machine, In the MMF harmonic spectrum in these cases there are also waves of order 5 and 7, absent, moreover, in the case of asymmetrical hexaphase winding.

3. Decreasing the magnitude of higher harmonics

The negative influence of higher harmonics, especially those of order 5 and 7 in the case of double-three-phase or symmetrical six-phase machines, can be reduced by applying the classical methods used in the construction of three-phase asynchronous machines [13, 18, 19]:

a) selection of the optimal number of sections q per pole and phase of the winding

b) the shortening of the opening y of the winding sections in relation to the diametral polar pitch τ, c) inclination of the slots on the rotor or stator.

3.1 Application of distributed winding

From relations (10) and (11) we can see a directly proportional dependence of the amplitude of the MMF harmonics in relation to the winding factor $k_{w.v}$, which is the product of the factors: distribution $k_{q.v}$, and shortening $k_{sc.v}$: $k_{w.v} = k_{q.v} \cdot k_{sc.v}$

The relation (18), derived for the calculation of the distribution factor [10,14,15], includes only one parameter that can be manipulated - the number of slots per pole and phase q in which the sides of the phase winding sections are arranged:

$$k_{q.v} = \sin\left(q\frac{vp\pi}{z_1}\right)\Big/\left(q\sin\left(\frac{vp\pi}{z_1}\right)\right) = \sin\left(v\frac{\pi}{2\,m}\right)\Big/\left(q\sin\left(v\frac{\pi}{2\,mq}\right)\right) \tag{18}$$

At the same time, it should be noted that increasing the number of phases m inevitably leads to a decrease in the possibilities of maneuvering with this parameter.

Table 3 shows the results of calculating the distribution factor for 6 types of windings for three-phase and six-phase machines, which differ by the number of phases m and the value of the parameter q. The scenario is considered: windings with diametral pitch, stator slots without inclination. Based on these data, important conclusions can be drawn that must be taken into account when designing multiphase machines:

a) in the case of windings with q=1, the distribution factor is unitary for the entire spectrum of harmonics;

b) for symmetrical three-phase and six-phase machines, windings with at least 3 notches per pole and phase can be recommended, which ensure a significant reduction of the most influential harmonics: 5, 7, 11 and 13;

c) in the case of hexaphase machines with asymmetrically arranged winding, a q=2 is sufficient to reduce harmonics 11 and 13. Harmonics 5, 7, 17, 19 are missing from the harmonic spectrum of the MMF, and those of order 23, 25 and above are insignificant as amplitude;

d) when applying distributed windings (q>1) the fundamental harmonic of the MMF is slightly reduced (⩽ 4.5%), which is easily compensated by a few extra turns in the phase winding.

Table 3

Change of the distribution factor according to the number of sections per phase and pole q and the order number of the harmonic ν. For case: windings with diametral pitch, stator notches without inclination

m	q	Harmonic order ν: 1	3	5	7	9	11	13	15	17	19	23	25
3	1	1	1	1	1	1	1	1	1	1	1	1	1
3	2	0.97	0.707	0.259	-0.259	-0.707	-0.966	-0.966	-0.707	-0.259	0.259	0.966	0.966
3	3	0.96	0.667	0.218	-0.177	-0.333	-0.177	0.218	0.667	0.960	0.960	0.218	-0.177
6	1	1	1	1	1	1	1	1	1	1	1	1	1
6	2	0.99	0.924	0.793	0.609	0.383	0.131	-0.131	-0.382	-0.609	-0.793	-0.991	-0.991
6	3	0.99	0.911	0.762	0.561	0.333	0.105	-0.096	-0.244	-0.323	-0.323	-0.095	0.105

Note: m – the number of phases.

3.2 Applying the pitch shortening procedure

The procedure of shortening the coils (winding sections) of phase y in relation to the diametral opening (y=τ) is applied to decrease the amplitude of only certain harmonics (one or two). In this case, it is handled with the y parameter of the winding but also with the winding mode (e.g. winding in two layers). The shortening factor is calculated with the relation [10,15]:

$$k_{sc} = \sin\left(\nu\, \beta_{sc} \frac{\pi}{2}\right), \tag{19}$$

where: $\beta_{sc} = y/\tau$ - is the relative value of the opening.

From (14) it follows that the harmonic with order number ν can be canceled if y satisfies the equation:

$$y = 2k\tau/\nu\,,\ \ k\ \in \mathrm{N} \tag{20}$$

If the solution is executable (y constitutes an integer number of notches), then the harmonic ν does not appear in the expression of the induced electromotive voltages. For example, for a relative step of 4/5, the 5th harmonic is excluded, and for a relative step of 6/7, the 7th harmonic is excluded. As a rule, the shortening is done in such a way as to decrease two neighboring harmonics to the same extent. For example, when selecting the step y/τ=0.83, the 5th but also the 7th harmonic is significantly reduced. In this context,

Figure 3 shows a parametric diagram of the relationship $k_{sc} = f(v, \beta_{sc})$, in which each curve refers to a spatial harmonic indicated in the menu.

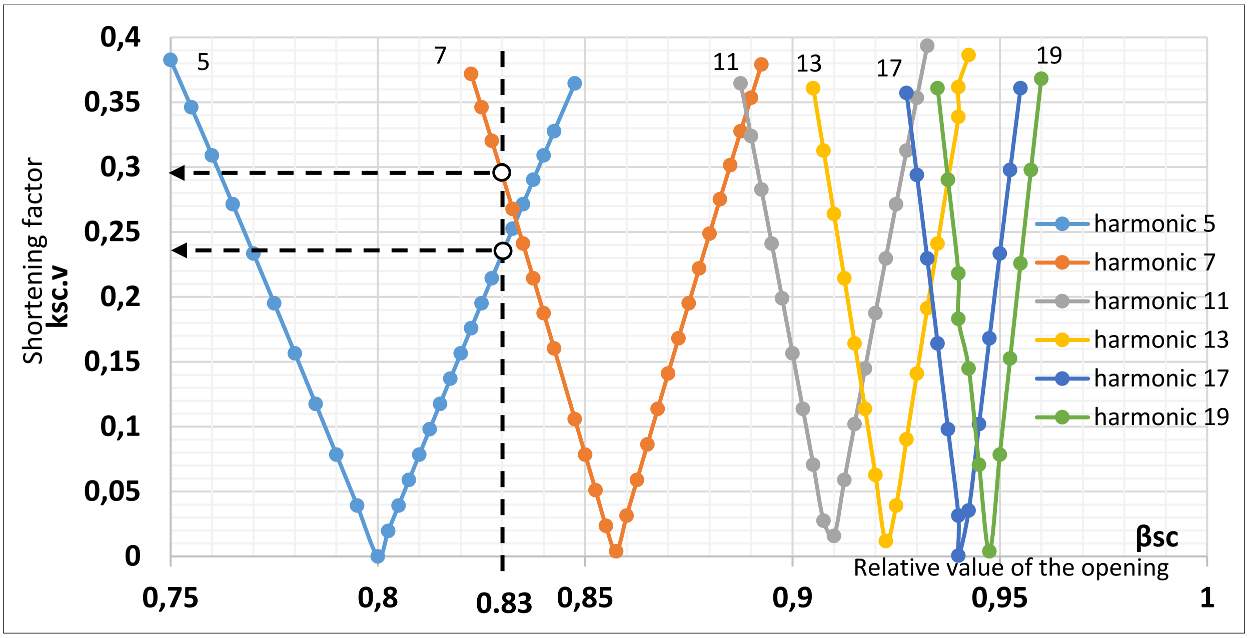


Figure 3. Shortening factor diagram.

3.3 Application of notch tilting procedure

At low and medium powers, the third method of reducing the amplitude of the higher harmonics of the MMF can be applied by tilting the rotor notches, if it is caged. The inclination factor expresses the degree of reduction of the MMF harmonic amplitude in relation to the inclination angle and is calculated with the relation [10,15]:

$$k_{in.v} = \frac{\sin v p \gamma}{v p \gamma} = \frac{\sin(b_r v p \pi / z_2)}{b_r v p \pi / z_2}, \tag{21}$$

where: $2\gamma = \frac{b}{t_c} \cdot \frac{2\pi}{z_2}$ - the angle of inclination of the notches / the displacement between the ends of the notches, $t_c = 2pR/z_2$ - pitch of notches, R- rotor radius, z_2 – the number of notches on the rotor, $b_r = b/t_c$ - the inclination expressed in steps of notches t_c.

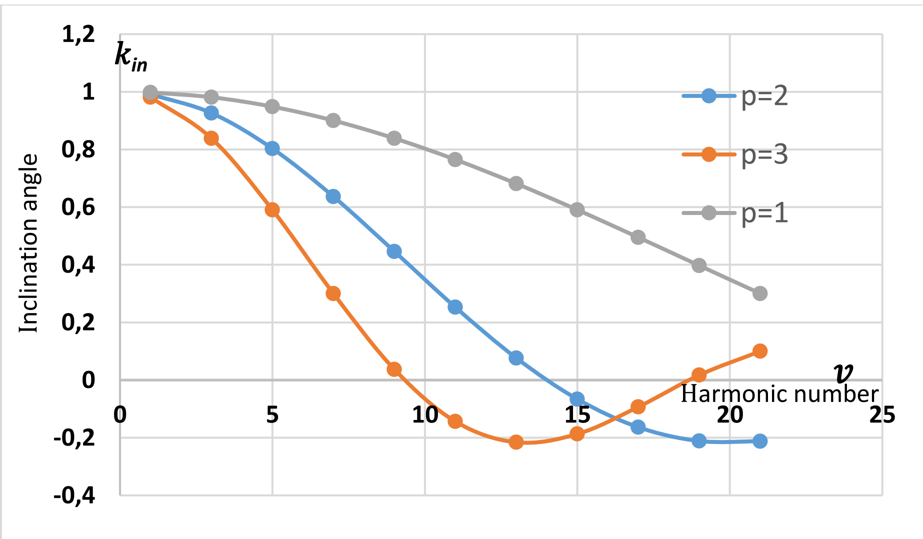


Figure 4. Chart of notch tilt factor $k_{in} = f(v)$,

Note: p - the number of pole pairs.

The inclination is usually done with a step or half a step of rotor notch. The efficiency of the tilting procedure can be seen by analyzing the diagram in Figure 4, where the $k_{in} = f(v)$, diagrams are presented, calculated for 3 variants of the number of pole pairs with $b_r = 1$ and $z_2 = 28$.

It is observed that tilting the notches by one tooth pitch leads, for example, in the case of 6-pole machines to practically complete suppression of the upper harmonics, starting with the seventh.

For a rotor winding, the winding factor will be defined with the relation: $k_{wr.v} = k_{qr.v} \cdot k_{in.v}$.

At high powers the tilting procedure is not applied due to considerable additional losses caused by the appearance of transverse (leakage) currents between the rotor bars [13,14]. In these cases, the focus is on shortening the step of the stator winding, choosing the number and optimal ratio of the notches on the stator and rotor.

4. Validation of results

4.1 Argumentation of the measurement methodology and harmonic analysis of MMF, induction, magnetic flux and induced electromotive voltage

As noted above, the MMF and air-gap magnetic induction wave, determined by a stator winding, carried by a sinusoidal current, contains, in addition to the fundamental harmonic, a series of higher harmonics. From relation (8) it follows that the weight of each harmonic can be appreciated by the ratio ($k_{wa.v}/\nu$) calculated for the winding, which determines the field (denoted for example by A). The winding factor $k_{wa.v}$ is calculated for the harmonic ν. Each harmonic of the magnetic induction in the gap causes / induces in a certain winding (denoted, for example, with B), an electromotive voltage. The effective value or its magnitude is weighted by the calculated ratio ($k_{wb.v}/\nu$). Here $k_{wb.v}$ is the winding factor calculated for winding B (in which the electromotive voltage is induced). Thus, the effective value or the amplitude of the electromotive voltage, induced in the winding B by the harmonic ν of the magnetic field in the gap, created by the winding A, is weighted by the ratio ($k_{wa.\nu} \cdot k_{wb.\nu}/\nu^2$).

In the case of the symmetrical six-phase machine the coils of the phase windings (one from each three-phase set) are arranged in the same slots. If a three-phase complete is powered, it will create a magnetic field in the gap, which induces in the second three-phase set electromotive voltages (EMV) identical to the self-induction electromotive voltages in the first set [6,20,21]. The harmonic structure of this voltage is also identical (with small deviations) to the harmonic structure of the air-gap induction and MMF. In this case $k_{wa.\nu} = k_{wb.\nu} = k_{w.\nu}$, and the weight of EMV harmonics can be estimated with the ratio ($k^2_{w.v}/\nu^2$).

Thus, the voltage induced in the secondary or self-induced in a stator phase can be defined with the relation:

$$e_f(\omega t) = E_{m.1}\sin\omega t \sum_{v}\left[\left(\frac{k'_{w.v}}{v}\right)^2 \cos\left(\nu\frac{\pi x}{\tau}\right)\right] \quad (22)$$

For an angle $\omega t = \pi/2$, x=0 at which all harmonics reach the value $\pm E_{m.v}$, it will be obtained:

$$E_{m.f} = E_{m1.f}\left[1 - \left(\frac{k'_{w.3}}{3}\right)^2 + \left(\frac{k'_{w.5}}{5}\right)^2 - \left(\frac{k'_{w.7}}{7}\right)^2 + \left(\frac{k'_{w.9}}{9}\right)^2 - \left(\frac{k'_{w.11}}{11}\right)^2 + \left(\frac{k'_{w.13}}{13}\right)^2 - \cdots\right] \quad (23)$$

In the case of asymmetric hexaphase machines, the axes of the three-phase sets are offset by 30° electrical, therefore the phase voltages in the secondary will have a reduced amplitude and a phase offset of (v ·30°).

4.2 Program of the experimental study

Taking into account the above, the following program of the experimental study were defined:

1. Taking the oscillograms of the line and phase voltages on a six-phase machine in induction mode with the aim of following the links between EMVs induced in the inductive (powered) and induced (secondary) phases of the six-phase machines in induction mode;

2. Taking the oscillograms of the line and phase voltages on the three-phase machines, their harmonic analysis to confirm the correctness of the calculation algorithm of the harmonic structure of the MMF in the gap of the induction machines with different number of poles and sections per pole and phase;

3. Taking the oscillograms of the phase voltages on the six-phase machines with symmetrically and asymmetrically arranged windings, the harmonic analysis of the phase voltages in order to determine the applicability of the developed calculation algorithm and the confirmation of the results of the analytical study on the harmonic structure of the MMF in the gap of n-three-phase machines;

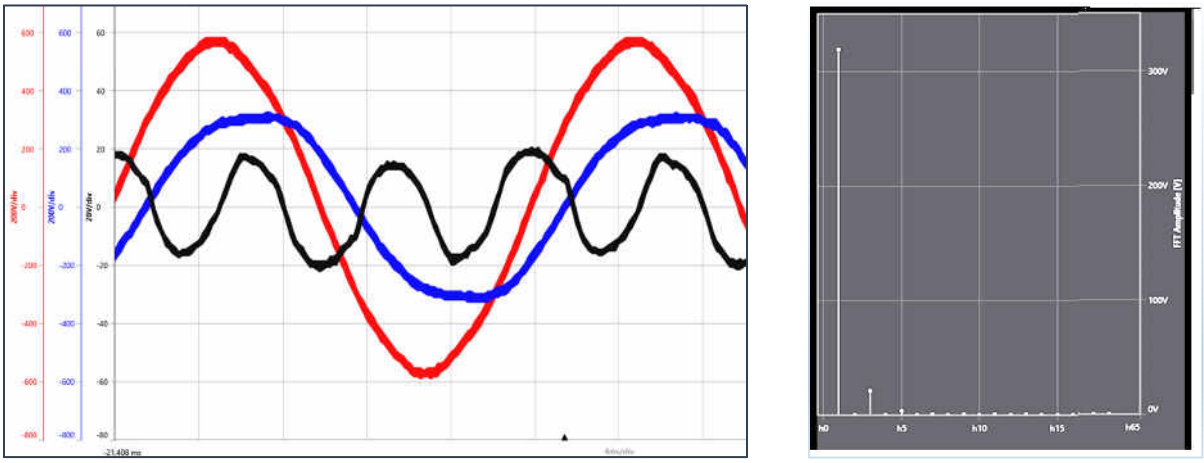
4. Taking the oscillograms of the voltages between the nodes of the three-phase sets of the windings in relation to the neutral wire of the power supply in order to obtain experimental arguments regarding the value and orientation of the phasors of the MMF harmonics of multiple order to 3.

The experimental tests were carried out on 4 induction machines, made on identical constructive samples but with different stator windings: MA3F-1500 and MA3F-1000 – three-phase machines with 4 and 6 poles, respectively; MA6F-SIM and MA6F-ASIM – 6-pole hexaphase machines, with windings arranged symmetrically and asymmetrically, respectively. The windings of the examined machines are executed with diametral pitch, y=τ. The machines' rotors are identical and each have 28 pitched notches.

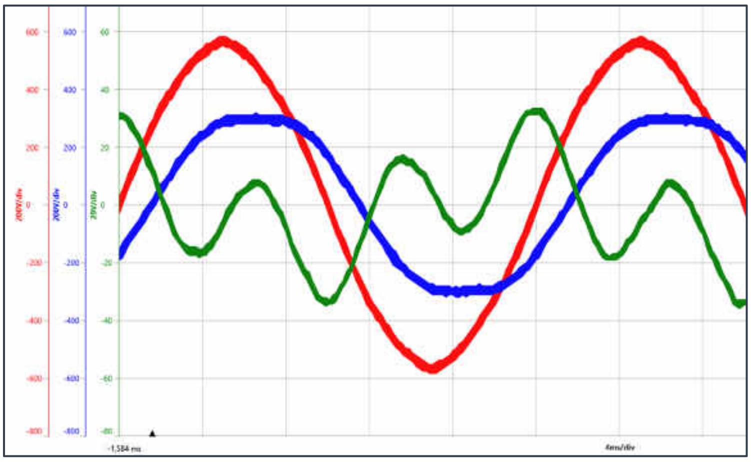
The basic instrument used for measurements and harmonic analysis – oscilloscope type FLUKE 190-204 SCOPOMETER 4CH 200MHz 2.5GS/s.

4.3 Analysis of the results of the experimental study

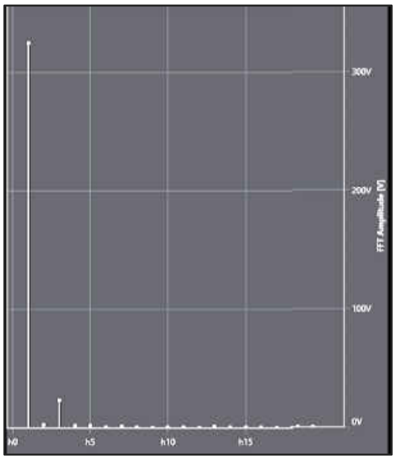
Figures 5, 6 and 7 show examples of oscillograms, taken in the idling regime of the experimental samples described above.



a) b)

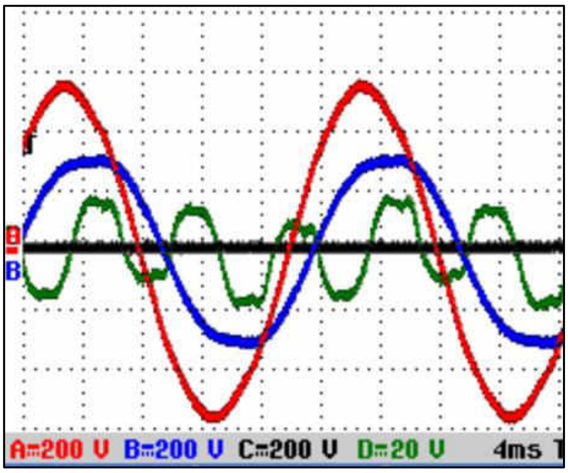


c)

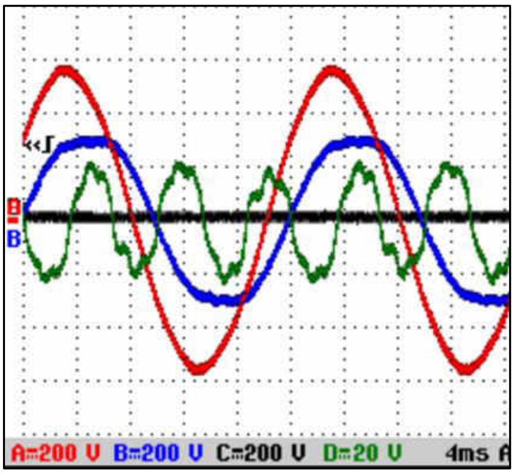


d)

Figure 5. The MA6F-SIM six-phase machine in induction mode: a) and c) – voltage diagram for the primary and secondary set respectively: red - line voltage, blue - phase voltage, green - node voltage; b) and d) - the harmonic component of the phase voltages of the primary and secondary sets, respectively.

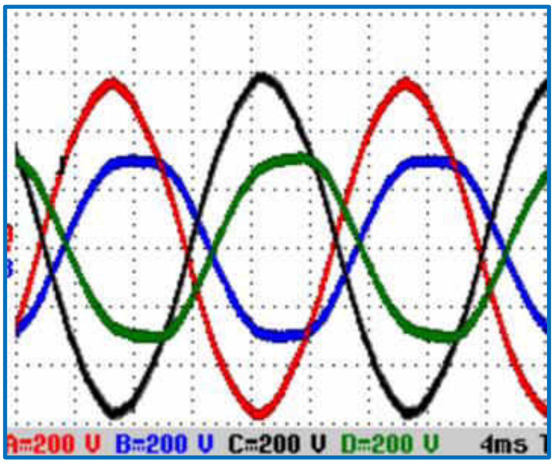


a)

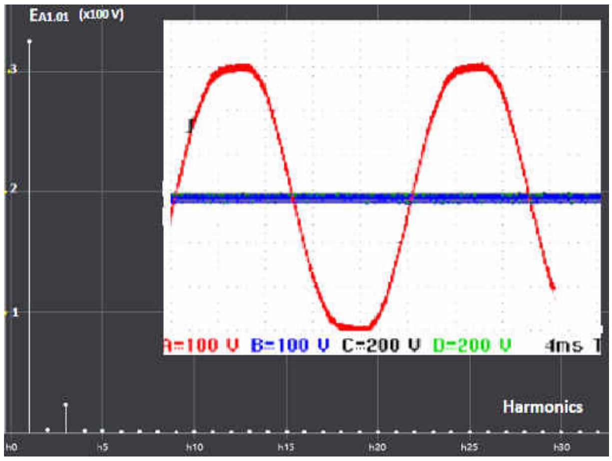


b)

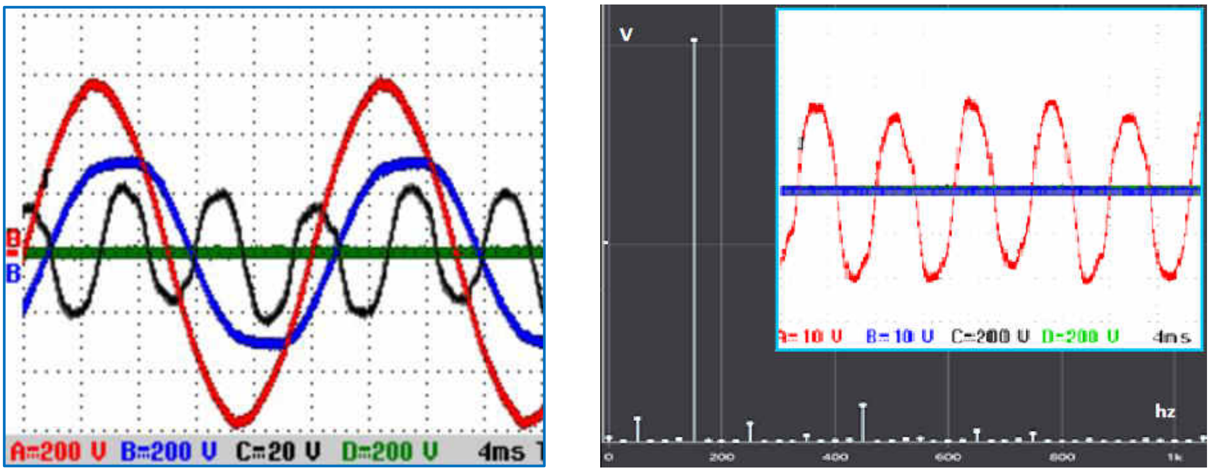
Figure 6. Voltage diagram: line (red), phase (blue) and node (green):
a) machine MA3F-1000 rpm, b) machine MA3F-1500 rpm.



a)



b)



c) d)

Figure 7. Voltage diagram and their harmonic structure. MA6F-SIM machine with 6-phase power supply, operating in idle mode: a) line and phase voltages: $E_{A1.C1}$-red, $E_{A1.01}$ - blue, $E_{B2.C2}$- gray , $E_{B2.02}$ – green; b) the harmonic structure of the phase voltage $E_{A1.01}$; c) voltage diagram of a three-phase set: $E_{A1.C1}$ - red, $E_{A1.01}$ - blue, $E_{01.02}$ - green, $E_{01.N}$ - gray; d) the harmonic structure of the voltage $E_{01.N}$

Table 4 shows the results of the harmonic analysis performed for each of the measured voltages, using the software provided with the FLUKE 190-204 oscilloscope. The table also includes the results of the calculations regarding the harmonic composition (TEM magnitude), based on the relations deduced above (18, 13, 14, 16).

Table 4

Harmonic structure of self-induction electromotive voltages in a phase winding

Order v	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**
Example 1	**MA3F: 2p=4, m=3, q=3, br=1, βsc=1, ksc=1**							
$k'_{w.v}$	1	0.694	0.227	-0.185	-0.347	-0.185	0.227	0.694
$E_{mf.v}$, calculating.	324	17.4	0.67	0.226	0.482	0.092	0.01	0.700
$E_{mf.v}$, measured	**324**	**20.2**	**2.37**	**1.78**	**1.78**	**0**	**0**	**0**
Example 2	**MA3F: 2p=6, m=3, q=2, br=1, βsc=1**							
$k'_{w.v}$	1	0.732	0.268	-0.268	-0.732	-1	-1	-0.732
$E_{mf.v}$, calculating	327.5	19.5	0.94	0.48	2.16	2.71	1.94	0.78
$E_{mf.v}$, measured	**327.5**	**19.6**	**4.2**	**1.2**	**1**	**2**	**1.2**	**0.7**
Example 3	**MA6F-SIM: 2p=6, m=6, q=2, br=1, βsc=1**							
$k'_{w.v}$	1	0.732	0.268	-0.268	-0.732	-1	-1	-0.732
$E_{mf.v}$, calculating.	**325**	**19.3**	**0.93**	**0.48**	**2.15**	**2.69**	**1.92**	**0.77**
$E_{mf.v}$, measured	**325**	**21**	**1.8**	**0.2**	**1.2**	**0.2**	**0.1**	**0.1**
Example 4	**MA6f-ASIM: 2p=6, m=6, q=1, br=1, βsc=1**							
$k'_{w.v}$	0.9812	0.8385	0.5904	0.3	0.03	-0.144	-0.221	-0.187
$E_{mf.v}$, calculating.	319.3	25.91	0	0	0.004	0.057	0.096	0.052
$E_{mf.v}$, measured	**319.3**	**21**	**2.0**	**0**	**0**	**0**	**0**	**0**

Note: v - harmonic number (order harmonic); $E_{mf.v}$ - the self-induction electromotive voltages; p - the number of pole pairs; $k'_{w.v}$ - the winding factor calculated for the harmonic v; MA3F - three-phase machines with 4 and 6 poles, respectively; MA6F-SIM - 6-pole hexaphase machines, with windings arranged symmetrically; MA6F-ASIM – 6-pole hexaphase machines, with windings arranged asymmetrically.

Based on the results of the experimental tests and those obtained by calculation (table 4) it is found:

- The harmonic structure of the voltages induced in the phase windings of the six-phase machine in induction mode is identical to that of the voltages induced in the second complete three-phase. Thus, for the assessment of the harmonic structure of self-induction and induced voltages, the measurements performed on any of these three-phase sets connected in a star with isolated nodes in relation to the neutral wire of the power supply can be used.
- The line voltages do not contain higher harmonics, they keep the form of the line voltage imposed by the power supply.
- The signal captured as a potential difference between the node of a three-phase set and the neutral wire of the power supply comprises the entire spectrum of multiple harmonics at 3, present in the phase voltage of the winding.
- In the case of machines with symmetrically arranged windings with six-phase power supply, the potential difference between the nodes of the two three-phase sets is zero, which confirms the fact that the phasors of the respective harmonics in all 6 phases coincide. In the case of machines with asymmetrically arranged winding, the rated voltages ($\mathrm{E}_{02.\mathrm{N}2}$ and $\mathrm{E}_{01.\mathrm{N}1}$) have the same magnitude but are out of phase by 30°.
- The results of the measurements, carried out on experimental samples, largely correspond to the quantities determined by calculations (according to the methodology described in the paper) regarding the harmonic structure of the MMF, the magnetic field created by it in the air gap of the six-phase machines, and the voltages induced in the phase windings.

5. Conclusions

1. The mathematical model of the magnetomotive force in the air gap of the six-phase asynchronous machine with various variants of the stator winding structure was developed.
2. Results of the measurements carried out on experimental samples confirm, to a large extent, the accepted hypotheses and the relations proposed for the calculation of the harmonic structure of the magnetic field in the air gap of the six-phase asynchronous machines.
3. It has been shown that the harmonic structure of the MMF of six-phase induction machines improves substantially compared to three-phase if the winding is arranged asymmetrically. The 5^{th}, 7^{th}, 17^{th}, 19^{th} order harmonics are suppressed.
4. The symmetrical six-phase induction machine from the point of view of the harmonic structure of the MMF in the air gap is identical to a three-phase machine.
5. As effective tools for reducing the magnitude of the higher harmonics (parasites) of the MMF in the air gap of the six-phase machine, the following can serve: the use of distributed windings, the shortening of the winding step, the inclination of the rotor notches, but also the selection of the optimal number of notches on the stator and rotor

The results of the present paper will be used in further research devoted to the study of asynchronous torques created by higher spatial harmonics in the air gap and additional loss reduction technologies in six-phase motors in healthy and fault operating regimes.

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