EQUATIONS OF THE HEART MODELING ROMANTIC ATTRACTION

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Abstract. *This article presents a mathematical model that focuses on the dynamics of romantic attraction and aims to stimulate interest in the areas of differential equations and mathematical modeling. Aimed at mathematicians, scientists and university students interested in differential equations, the study offers new insights into the complex dynamics of romantic relationships from a mathematical perspective and challenges conventional ideas about attraction and commitment. We introduce the notion of differential equations to capture the evolution of romantic attachments over time and conducting mathematical analysis and computer simulations. The outcomes indicate that employing mathematical models can offer a more profound understanding of human behavior and encourage interdisciplinary collaboration in education and ongoing research where mathematics and social sciences intersect. This perspective enriches understanding of romantic relationships and stimulates thought about broader applications in various real-world contexts. Further exploration is needed to refine the mathematical model and explore its practical applications in real-world contexts.*

Keywords: *attachment styles, computer modeling, eigen values, eigen vectors, solving linear differential equations*

Introduction

It is likely that you are familiar with "love meters" – websites that provide a percentage of compatibility between two people when provided with their names, birthdays and other information. While their scientific validity is highly uncertain they exemplify the intersection of mathematics and social sciences in popular culture. Despite the perception of mathematics and social sciences as separate and non-interfering domains, mathematical principles can yield rich insights into human behavior and social phenomena.

After Strogatz [1] applied a system of linear differential equations to study romantic dynamics in his one-page work (1988) and later in his book (1994) the topic has gained attention among many researchers. With the introduction of mathematical models, it became possible to simulate different scenarios how the dynamics would change over time, one such model on which this article is partially based on was implemented by Dabler [2].

What types of romantic styles attract? Do opposites fall in love with each other? How do individuals with different emotional attachment interact and influence each other's feelings? By examining different attachment styles and proposing ways to model them, we aim to answer these questions.

The model

We will start by introducing Romeo. Consider the real-valued function $r(t)$ defined by the real parameter t which denotes the time passed. The value $r(t) > 0$ represents the love, for $r(t) = 0$ - neutral state, and for $r(t) < 0$ the dislike of Romeo. Let's consider the simplified case where Romeo's affection changes based on his own feelings. This can be described by the following ordinary differential equation (ODE):

$$\frac{dr}{dt} = ar \tag{1}$$

Where a is a real-valued coefficient that represents the attitude of Romeo, when $a>0$ Romeo tries to amplify his feelings, for $a = 0$ the rate of change of his feelings remains constant, for $a<0$ the rate of change heads in the opposite of what Romeo feels at the moment t. From (1) we can obtain that the general solution is:

$$r(t) = ke^{at} \tag{2}$$

where $k \in \mathbb{R}^*$, for $r(0) = 0$, the solution extends to $r(t) = ke^{at}$ where $k \in \mathbb{R}$.

Let us consider another example, let's say that instead of a constant real-valued parameter a the attitude of Romeo is described by a function $f(t)$, that is Romeo's attitude changes over time. Let $f(t)=A\sin\lambda t$, which means that Romeo's attitude changes over time periodically with frequency λ and amplitude A. We obtain the following equations:

$$\frac{dr}{dt} = Ar\sin\lambda t \tag{3}$$

$$r(t) = ke^{-A\lambda\cos\lambda t}, k \in \mathbb{R} \tag{4}$$

Where equation (4) is the general solution of (3). Knowing the initial condition $r(0)$, we can derive $k = r(0)\, e^{A\lambda}$. Consider the following Initial Value Problem with $A = 2$, $\boldsymbol{\lambda} = 1$ and his feelings are positive with value 1 at $t = 0$. Solving for $r(0) = 1$, we obtain that $k = e^2$ thus, the solution to this IVP is

$$r(t) = e^{-2\cos\boldsymbol{t}+2} \tag{5}$$

For $r(0) = -0.5$ from (3) we obtain $k = -0.5e^2$ the equation becomes

$$r(t) = -0.5e^{-2\cos\boldsymbol{t}+2} \tag{6}$$

For $r(0) = 0$ we obtain $k = 0$ the equation becomes the constant function $r(t) = 0$. The graphs (Red: $r(0) = 1$; Green $r(0) = -0.5$) of the equation (4) with $A = 2, \lambda = 1$ are presented in Figure 1.

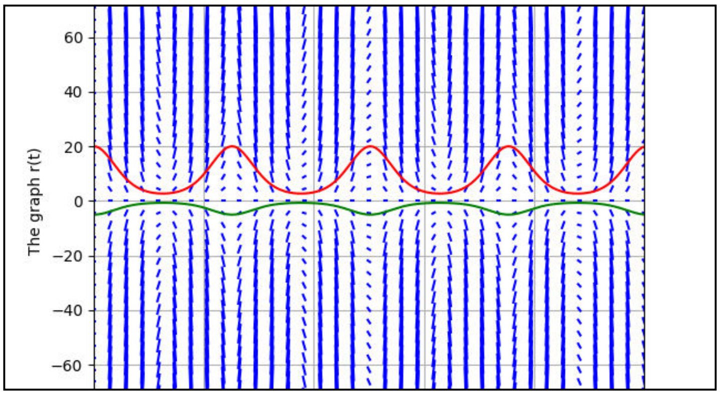

Figure 1. The slope field with the given initial conditions

From Figure 1, we can conclude that Romeo amplifies his feelings and then dampens his feelings periodically over the period $T = 2\pi$ Romeo reaches his initial state. Generally, the set of possible solutions can be visualized using a slope field.

The equations discussed so far are examples of differential equations where the rate of change in time of Romeo's feelings depends on his feelings. Introducing Juliet, we can model the interaction between Romeo's feelings for Juliet and Juliet's feelings for Romeo by the use of the following system of constant coefficients linear differential equations:

$$\begin{cases} \frac{dr}{dt} = \alpha r + \beta j \\ \frac{dj}{dt} = \gamma r + \delta j \end{cases} \tag{7}$$

$$\frac{dx_1}{dt} = ax_1 + bx_2 \tag{8}$$

Where α, β, γ, δ are constants and r and j are functions of time expressing the feelings of Romeo and Juliet. We may write the coefficients α, β, γ, δ from system (7) in a matrix and rewrite the system as:

$$A\begin{pmatrix} r \\ j \end{pmatrix} = \begin{pmatrix} r' \\ j' \end{pmatrix} \tag{9}$$

Where $r' = \frac{dr}{dt}$ *and* $j' = \frac{dj}{dt}$. $A = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$. Thus, rewriting in this form allows us to find the eigenvalues $\lambda_{1,2}$ and the eigenvectors of (9). In order to compute the eigenvalues, we write the matrix $\begin{pmatrix} r \\ j \end{pmatrix}$ as $\vec{x}$ and $\begin{pmatrix} r' \\ j' \end{pmatrix} = \dot{\vec{x}}$. Now we need to find a value λ such that $A\vec{x} = \lambda\vec{x}$, therefore:

$$(A - \lambda I)\vec{x} = 0 \tag{10}$$

Equation (10) is true if and only if $det(A - \lambda I) = 0$, we have that:

$$\lambda^2 - (\alpha + \delta)\lambda + \alpha\delta - \beta\gamma = 0$$

$$\lambda_{1,2} = \frac{(\alpha+\delta) \pm \sqrt{(\alpha+\delta)^2 - 4det(A)}}{2} \tag{11}$$

Assuming λ_1, λ_2 are real valued, we may consider a point (x, y) described by a vector $\vec{x}$ in the phase space of $r(t), j(t)$, then we can compute the first eigenvector as:

$$A\begin{pmatrix} x \\ y \end{pmatrix} = \lambda_1 \begin{pmatrix} x \\ y \end{pmatrix} \tag{12}$$

From (13) we obtain the following linear system:

$$\begin{cases} \alpha x + \beta y = \lambda_1 x \\ \gamma x + \delta y = \lambda_1 y \end{cases}$$

With the set of solutions:

$$y = \frac{\lambda_1 + \gamma - \alpha}{\lambda_1 - \delta + \beta} x \tag{13}$$

Therefore, we can choose the arbitrary eigenvectors as follows:

$$\overrightarrow{\eta_1} = \begin{pmatrix} 1 \\ \frac{\lambda_1 + \gamma - \alpha}{\lambda_1 - \delta + \beta} \end{pmatrix}; \overrightarrow{\eta_2} = \begin{pmatrix} 1 \\ \frac{\lambda_2 + \gamma - \alpha}{\lambda_2 - \delta + \beta} \end{pmatrix};$$

If equation (11) has no real roots then there are no eigenvectors, and if it yields only one real-valued root, then it has only one eigenvalue that can be computed using formula (13).

The subsequent paragraphs will attempt to present some hypothetical examples that make use of the system discussed so far for modeling love attachment styles. Using the model of adult attachment theory proposed by Lawrence Robinson, Jeanne Segal and Jaelline Jaffe [3], we can categorize attachment as one of four main styles: Secure, Preocupied, Avoidant and Disorganized.

Secure attachment style indicates that an individual is feeling secure within the relationship, being comfortable to express their emotions and needs, and providing support to their partner. Considering equation (8) for reference, if an individual x_1 has a secure attachment style it can be modeled by setting the coefficient to have a negative value a<0 and b<0. We can model an individual with a preoccupied attachment style by considering a<0 and b>0. An individual with an avoidant insecure attachment style prioritizes self-reliance, often avoiding closeness and intimacy in a relationship, we may consider the coefficients as a>0 and b<0. A person who has a disorganized attachment style may be modeled using a>0 and b>0.

Those with a disorganized insecure attachment style show inconsistent behavior, swinging between extremes of love and hate towards their partner, they often suppress their true feelings. The affection swings can be modeled using a periodic function similar to the equation (3).

For instance, consider the following system of differential equations:

$$\begin{pmatrix} -2 & -1 \\ -0.2 & -2 \end{pmatrix} \vec{x} = \dot{\vec{x}} \tag{14}$$

The system (14) has coefficients $\alpha = -2, \beta = -1, \gamma = -0.2, \delta = -2$, which means that Romeo and Juliet both have secure attachment styles. We can compute the eigenvalues as $\lambda_{1,2} = -2 \pm \frac{\sqrt{5}}{5}$, both of the eigenvalues are real-valued and negative, meaning that the solution tends to an equilibrium point $(0,0)$, according to the criteria for critical points [4, pp. 148-151], that is independent of the initial conditions $r(0)$ and $j(0)$. In Figure 2 is presented the phase space of equation (14) with the grey lines indicating the position of the eigenvectors, and the color bar representing the vectors' magnitude of the phase space.

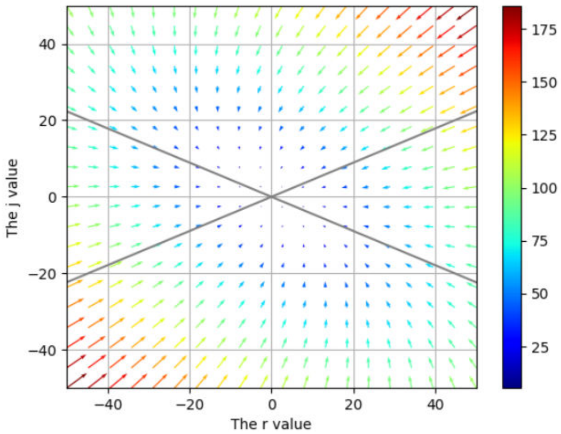

Figure 2. Phase space of equation (14)

The general solution to the system of equations (7) can be written in the following form [4, p. 141]:

$$\vec{x} = c_1\overrightarrow{\eta_1}e^{\lambda_1 t} + c_2\overrightarrow{\eta_2}e^{\lambda_2 t} \tag{15}$$

Where c_1, c_2 are real valued coefficients, $\overrightarrow{\eta_1}, \overrightarrow{\eta_2}$ are the corresponding eigenvectors of the eigenvalues λ_1, λ_2.

Conclusions

In summary, this research has built a model, where the mathematics of differential equations and rationalizations from attachment theory have been included in the dynamics of romantic attraction. The second part of the model which draws from the attachment theories of Lawrence Robinson, Jeanne Segal, and Jaelline Jaffe shows how the emotions of individuals affect their romantic connections. Using linear differential equation with constant coefficient to deduce the scenes that describe the various attachment styles, the complicated dyads of compassion among the couples were shed into light. The findings emphasize the importance of interdisciplinary strategies in studying human behavior as well as the extent to which mathematical modeling enlarges the possibilities to generate unique insights from complex societal processes. Besides that, the findings of this research reveal the applicability of attachment theory for shedding light on the nuances of romantic ties, paving the way for further exploration and refinement of the proposed model. Basically, this study is valuable for providing new insights about mathematics and social sciences in the area in which the two disciplines overlap, provoking interdisciplinary collaboration and deepening the understanding of how people fall in love.

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