THE BEHAVIOR IN TIME OF THE BEAM PROPPED ON THE DEFORMABLE MEDIUM

Viorica Ţibichi
Technical University of Moldova

INTRODUCTION

Reviewing normal exploitation of the building, the great importance, is the study of its behavior in time. Normal operation can be ensured if is known the behavior under load of the building in question and evaluation of displacements and deformations over time of entire construction or its components.

Deformation of bodies will stabilize by achieving the equilibrium between internal and external forces. A change of equilibrium state will lead to new results. Safety and hazard-free operation is achieved by studying and systematic tracking of displacements and deformations of construction.

1. THE PROBLEM FORMULATION

Considering the case, that a given beam is resting on viscoelastic medium (fig. 1), stressed by two concentrated forces.

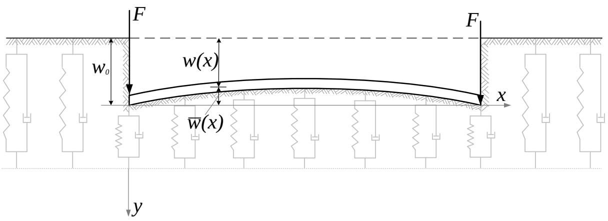

Figure 1.

For modeling of behavior of viscoelastic medium can be chosen different rheological models, which represent combinations of Maxwell and Kelvin simple models.

For beam in question is chosen that it rests on viscoelastic Kelvin-Voigt medium.

2. SOLVING THE PROBLEM

Given the differential equation of beam deformed axis:

$$\frac{d^2w}{dx^2} = -\frac{M}{EI}, \tag{1}$$

and reactive pressure of Kelvin-Voigt medium (fig. 2), resulting from the interaction between the beam and the environment:

$$p = kw + \eta\frac{dw}{dt}, \tag{2}$$

results differential equation:

$$\frac{d^2M}{dx^2} = kw + \eta\frac{dw}{dt}, \tag{3}$$

where: p – environment reactive pressure;
k – stiffness coefficient of the environment;
η – coefficient of viscosity of the environment;
w – displacement;
$\frac{dw}{dt}$ – speed of displacement.

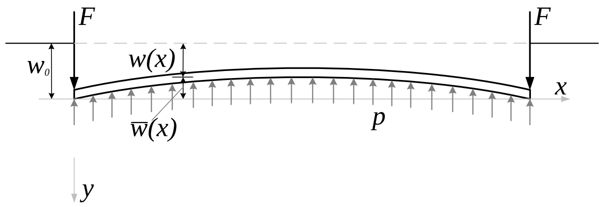

Figure 2.

Expressing the displacements of the sections of the beam according to the displacement of its extremity that is positioned in the origin of the coordinate system:

$$w(x,t) = w_0(t) - \overline{w}(x,t), \tag{4}$$

and taking account of (1) and (2) resulted differential equations:

$$\frac{d^2M}{dx^2} = kw_0 - k\overline{w} + \eta\frac{dw_0}{dt} - \eta\frac{d\overline{w}}{dt}. \tag{5}$$

System (5) is expanded in series. It contains five unknowns, and three equations more will be evolved. In relation to the time, the system is solved by successive approximations.

Considering that the speed of deformation of the starting section $\frac{dw_0}{dt}$ is constant over the time interval Δt, and its equal with semi-sum of values from the range of end sections, can be written:

$$w_{0,i+1} = w_{0,i} + \frac{\Delta t}{2}\left(\dot{w}_{0,i} + \dot{w}_{0,i+1}\right). \qquad (6)$$

Therefore, considering displasment of initial section over time i and time interval Δt, may be determined displacement of the initial section at the time $(i+1)$...

Entering the boundary conditions:

$$\text{for } i = 0 \quad \begin{cases} w_0 = \dfrac{dw_0}{dt} = 0 \\ \overline{W}_{n,i} = \dfrac{d\overline{W}_{n,i}}{dt} = 0 \end{cases}, \qquad (7)$$

result:

$$\begin{cases} \overline{w}_1 = \sum\limits_{n=1,3,5,...}^{\infty} \overline{W}_{n,1} \sin\left(\dfrac{n\pi x}{L}\right); \\ w_1 = w_{0,1} - \overline{w}_1; \\ M_1 = \sum\limits_{n=1,3,5,...}^{\infty} M_{n,1} \sin\left(\dfrac{n\pi x}{L}\right). \end{cases} \qquad (8)$$

And for $i = 1$ results $\overline{W}_{n,2}$, $w_{0,2}$ and $M_{n,2}$, knowing the previous step. Thus, determine $\overline{w}_2$, w_2, M_2 and … $\overline{w}_n$, w_n, M_n.

3. CALCULATION EXAMPLE

Was calculated a concrete reinforced beam ($E = 0{,}21 \cdot 10^5\, MPa$) with $40 \times 60\, cm^2$ section and $L = 3{,}6\, m$ length, with free ends, and loaded with loads $F = 100\, kN$ (fig. 1). The beam rests on the viscoelastic Kelvin-Voigt environment with characteristics: $E_0 = 200\, MPa$; $k = 4750\, N/m$, $\eta = 74 \cdot 10^{16}\, P$.

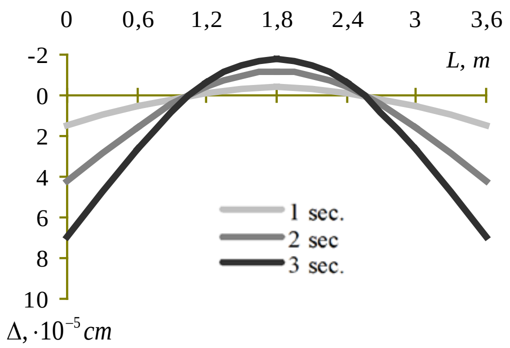

Figure 3. Evolution of the displacements in the sections of the beam.

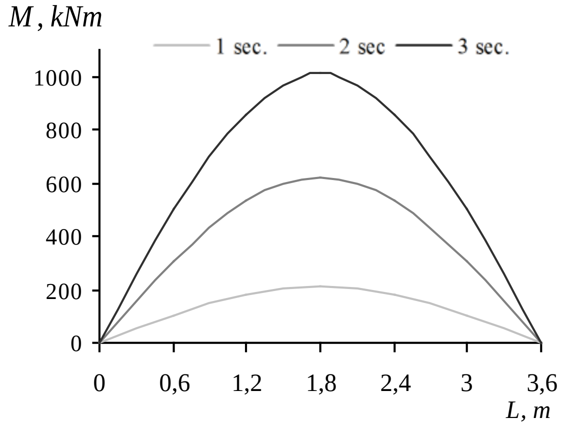

Figure 4. Development of the bending moments in the first three seconds.

The results are plotted in figures 3 and 4. Figure 3 shows the evolution of the displacements in the sections of the beam, in the first three seconds, and figure 4 shows the development of the bending moments.

This method allows the determination of the displacements and the bending moments in all sections of the beam at the given intervals of time.

Bibliography

1. Moczo P., Kristek J., Franek P. *Lecture notes on rheological models// Comenius University, Bratislava, 2006.*

1. Vasani P.C. *Interactive analysis models for soil and structures// L. D. College of Engineering Ahmedabad, http://www.fineprint.com.*

1. Ţibichi V. *Contribuţii privind evaluarea interacţiunii statice şi reologice la grinzile rezemate pe medii deformabile// Teză de doctorat, UT Gh. Asachi, Iaşi, 1999.*

2. Ungureanu N., Silion T., Gorbănescu D. *Grinzi pereţi rezemate pe suport deformabil// Buletinul I.P. Iaşi, Tomul XXI (XXV), Fasc. 3-4, Sec. V Construcţii. Arhitectură,1975.*

Recommended for publication: 12.09.2017.