Verification of system nets

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Abstract **— This paper provides the central basic of the modeling technique: the concept of system nets and there verifications. A formal framework for system nets has to establish the relationship between syntactical inscriptions (terms), at arcs and places, and their concrete semantical denotation. This relationship of syntax and semantics is mathematically well established, belonging to the basic concepts of computer science.**

Index Terms **— Petri net, place weight, sort, system net, structure, universe.**

I. INTRODUCTION

The idea to concisely represent elementary system net is: firstly, the local states are partitioned into several classes. Each class is then "folded" to a single place. Such a place contains particular items, representing the local states in the corresponding class. Likewise, the actions are partitioned into classes, and each class is folded to a single transitions. Inscriptions of the adjacent arcs describe the actions in the corresponding class. A distinguished action can be regained by evaluating the variables involved (by replacing them with constants).

II. SYSTEM NET

The introduction above explain the conceptual idea of system nets: each place of a system net Σ represents a set of local states and each transitions of Σ represents set of actions. The sets assigned to the places from the underlying universe A.

1. Definition Let Σ be a net. A universe A of Σ fixes for each place $p \in P_\Sigma$ a set A_p, the domain of p in A.

2. Definition Let Σ be a net with universe A.

i. A state a of Σ assigns to each place $p \in P_\Sigma$ a set $a(p) \subseteq A_p$.

ii. Let $t \in T_\Sigma$. An action m of t assigns to each adjacent arc $f = (p, t)$ or $f = (t, p)$ a set $m(f) \subseteq A_p$.

To explain definition above consider a system which consist of subsystems which share scarce resources. Such a resource is accessible by at most one component simultaneously. E.W. Dijkstra illustrated in [1] this system by "philosopher" and "forks" which stand for subsystems and resources:

"Five philosophers, are living in a house where the table is laid to them, each philosopher having his own place at the table. Their only problem – is that the dish served is a very difficult kind of spaghetti, that has to be eaten with two forks. There are two forks next to each plate, so that presents no difficulty, as a consequence, however, no two neighbors may be eating simultaneously ."

Figure 1. shows the Petri net which represent these system.

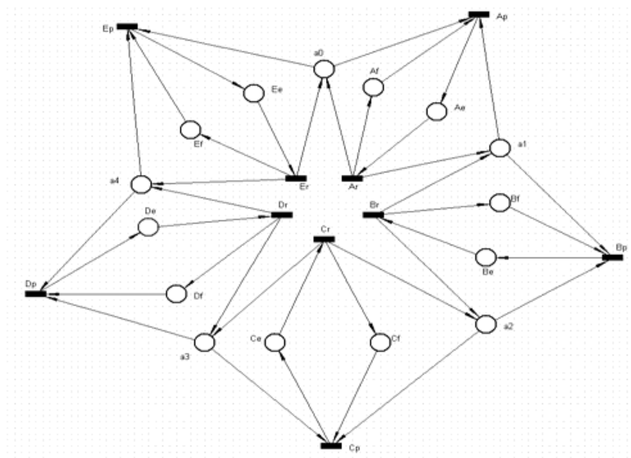

Fig. 1

We strive a more concise representation of the system by exploiting its regular structure. The essential idea is to represent a set of local states with similar behavior as a single place and likewise a set of action with similar behavior as a single transition. As a example, the five local states "a is thinking", "b is thinking", "c is thinking", "d is thinking", "e is thinking" may be assigned the place "thinking philosophers". Figure 2 shows corresponding system net: the local states of net are now clustered into three places: "thinking philosophers", "available forks", "eating philosopher", and two transitions: "picks up", "return". The instance of a distinguished action is in the folded version represented by an assignment of concrete items to the variables occurring at the surrounding arcs. In these way we obtained a more abstract and general representation.

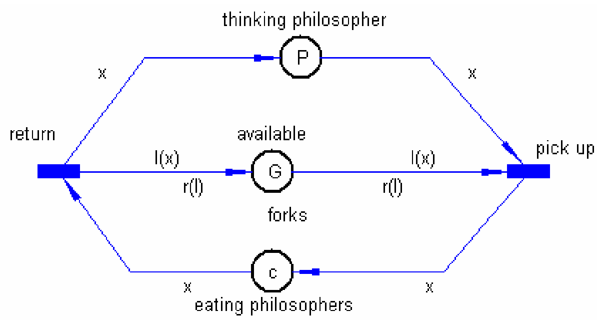

Fig. 2

$P = \{a, b, c, d, e\}$;

$G = \{f_1, f_2, f_3, f_4, f_5\}$, l, $r : P \rightarrow G$, x: variable over P;

$l(a) = l(b) = f_1$,
$l(b) = l(c) = f_2$,
$l(c) = l(d) = f_3$,
$l(d) = l(e) = f_4$,
$l(e) = l(a) = f_5$.

3. Definition Let Σ be a net with some universe A, let a be a state, let $t\in T_\Sigma$ and let m be an action of t.

1. m is enabled at a iff for each place $p\in{}^\bullet t$, $m(p, t) \subseteq a(p)$ and for each place $p\in t^\bullet$, $(m(t, p) \setminus m(p, t)) \subseteq A_p \setminus a(p)$.
2. the state $eff(a, m)$, defined for each place $p\in P_\Sigma$ by

$$eff(a,m)(p) := \begin{cases} a(p)\setminus m(p,t) & iff\ p\in{}^\bullet t\setminus t^\bullet, \\ a(p)\cup m(p,t) & iff\ p\in t^\bullet\setminus{}^\bullet t, \\ (a(p)\setminus m(p,t))\cup m(p,t) & iff\ p\in t^\bullet\cap{}^\bullet t, \\ a(p) & oterwise, \end{cases}$$

Is the effect of m's occurrence on a.

3. Assume m is enabled at a. Then the triple $(a, m, eff(a, m))$ is called a step of t in Σ, and usually written $a \xrightarrow{m} eff(a,m)$.

4. Definition A net Σ is a system net iff:

i. For each place $p\in P_\Sigma$, a set A_p is assumed (i.e. a universe of Σ),
ii. For each $t\in T_\Sigma$, a set of action of t is assumed,
iii. A state a_Σ is distinguished, called the initial state of Σ,
iv. Each transition $t\in T_\Sigma$, is denoted as either progressing or quiescent,
v. Some progressing transitions are distinguished as fair.

System nets [4, 5] have been represented above by means of sorted terms. Such terms ground on structures.

5. Definition Let $A_1,...,A_k$ be sets.

i. Let $a\in A_i$ for some $1\le i\le k$. Then a is called a constant in the sets $A_1,...,A_k$, and A_i is called a sort of a.
ii. For $i=1,...,n+1$, let $B\in\{A_1,...,A_k\}$, and let $f: B_1\times...\times B_n \to B_{n+1}$ be a function. Then f is called a function over sets $A_1,...,A_k$. The sets $B_1,...,B_n$ are the argument sorts and B_{n+1} is the target sort of f, The $n+1$-tuple $(B_1,...,B_n)$ is the arity of f and is usually written $B_1\times...\times B_n \to B_{n+1}$.

For example above b is the constant in P and G of sort P. Furthermore, l is a function over P and G with one argument sort P and the target sort G. Its arity is $P\to G$.

6. Definition Let $A_1,\ldots, A_k$ be sets, let $a_1,...,a_l$ be constants in $A_1,..., A_k$ and let $f_1,...,f_m$ functions over $A_1,\ldots, A_k$. Then $A = (A_1,...,A_k; a_1,...,a_l; f_1,...,f_m)$ is a structure. $A_1,\ldots, A_k$ are the carrier sets, $a_1,...,a_l$ the constants, and $f_1,...,f_m$ the function of A.

The system net which describes the philosophers system are based on structures. The structure for these system is $Phils = (P, G, a, b, c, d, e, f_1, f_2, f_3, f_4, f_5; l, r)$. Hence this structure has two carrier sets, ten constants, and two functions.

III. STATE PROPERTIES

State properties [2] are essentially based on weighted sets of tokens, formally given by multiset valued mapping on the places domains.

7. Definition Let Σ be a net with some universe A, let $p\in P_\Sigma$ and let B any multiset. The mapping $I: A_p \to B$ is a place weight of p. I is natural if $B = N$.

Place states [1, 3] weights can be used to describe invariant properties of system nets by help of equations that hold in all reachable.

8. Definition Let Σ be a net with some universe A, let B any multiset and let $P = \{p_1,\ldots, p_n\} \subseteq P_\Sigma$. For $j = 1,\ldots,k$, let $I^j : A_{pj} \to B$ be a place weight of p_j.

i. $\{I^1,\ldots,I^k\}$ is a Σ invariance with value B if for each reachable state s of Σ, $I^1(s(p_1)) + \cdots + I^k(s(p_k)) = B$.
ii. A Σ invariance $\{I^1,\ldots,I^k\}$ is frequently written as a symbolic equation $I^1(p_1) + \cdots + I^k(p_k) = B$ and this equation is said to hold in Σ.

In a Σ equation $I^1(p_1) + \cdots + I^k(p_k) = B$, the value of b is apparently equal to $I^1(s_\Sigma(p_1)) + \cdots + I^k(s_\Sigma(p_k)) = B$, with s_Σ the initial state of Σ.

The philosophers system has three euations:

$$A + C = a + b + c + d + e$$
$$B + l(C) + r(C) = f_1 + f_2 + f_3 + f_4 + f_5$$
$$r(A) + l(A) - B = f_1 + f_2 + f_3 + f_4 + f_5,$$

Where A- is place thinking philosophers,
C- is place eating philosophers,
B- is place available forks.

State properties can be proven by means of equations and inequalities such obtained above.

IV. CONCLUSION

State properties was considered in these paper. These properties can be proven by equations and inequalities, which in turn can be derived from the static structure of a given system net, by analogy to equations ad inequalities of elementary nets. Each place of the net serve as a variable, ranging over the subsets of the places domains.

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