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Active Filter on RC element with Distributed Parameters Sensitivity Analyze

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***Abstract*—This article presents the results of research of the stability of characteristics of the active filter on element with distributed parameters through the calculate the sensitivity of the Amplitude - Frequency Characteristic (AFC) of the filter on change the value of each element of the electronic circuit of active filter.**

***Keywords*— Active Filler, RC element with distributed parameters, Amplitude - Frequency Characteristic, transfer function .**

I. INTRODUCTION

The characteristics of transmission and reception channels in telecommunications and information networks are formed on the basis of diverse types of active filters, made on various active and passive elements. Active filters based on RC elements with distributed parameters (turned to be most effective $\overline{RC} - elements$).

$\overline{RC} -$ elements represent some elements, which contain in a single volume a structure (a component), which has parallel resistor properties with resistance R and capacitor properties with capacity C. These elements can be obtained based on several technologies, such as be the technology of thin films or the method of obtaining resistive microwire with glass insulation according to the Taylor-Ulitovschi technology and the coaxial microwire based on it. By choosing the composition of the resistive material, the dielectric and the composition of the conductive layer material with approximately equal values, but opposite in sense (meaning positive and negative), very high stabilities are obtained in the very high temperature band and time of the 10exp(-6) level 1/degree. These elements also have a high priority: they make it possible to greatly reduce the volume and mass of the electronic devices in which they are used, and relative decrease the energy consumption and the price of the devices.

The basic peculiarity of all electronic circuits based on $\overline{RC} -$ elements consists in the fact that all their functions are described by transcendental equations, in which the argument of the variable

$$\Theta = \sqrt{p\tau} = \sqrt{pRC}$$

represents an irrational function of a complex variable. This fact limits the possibilities of direct use to these circuits with $\overline{RC} -$ elements of the classical theory of analysis and synthesis of circuits with non-distributed elements. Therefore, when calculating the characteristics of the devices based on $\overline{RC} -$ elements arises the problem of presenting the transfer functions of such devices in a rational form. It is necessary to mention that the rational form, found as a result of solving the approximation problem, only approximately describes the characteristics of circuits based on $\overline{RC} -$ elements. Therefore, in order to evaluate the authenticity of the results of the analysis and synthesis of the mentioned circuits, it is necessary to know the degree of difference (error) of the approximate function compared to the exact transcendental function of the circuit based $\overline{RC} -$ elements. At present, several methods are used to solve this problem.

The trivial method consists in approximating the hyperbolic functions in Maclauren series and when limiting the series with two terms the error does not exceed 10%, and when limiting the series with three terms the error does not exceed 4% [1].

Another method consists in decomposing the hyperbolic functions into continuous series and depending on the required accuracy of the calculations to limit the number of terms of the series. For example, if only two terms of such a series are used, then the error of the approximation of the frequency characteristics for the passband of the filters does not exceed the value of 8%, and when the terms of the decoupling series increase, the approximation error suddenly drops. In the work given to perform the approximate analysis of the circuits based on $\overline{RC} -$ elements the decomposition of hyperbolic functions into continuous series of two or three terms was used [2].

It is necessary to mention that $\overline{\boldsymbol{RC}}$ – elements in various electronic circuits they can be connected in several variants of dipole or tripole and depending on this, a decomposition of the hyperbolic functions with a smaller or larger number of terms of the approximating series may be required in order not to exceed the predetermined approximation error.

The filters in the composition of telecommunications and information networks operate in different climatic conditions and therefore are subject to the action of various types of destabilizing factors such as: temperature, humidity, radiation, natural degradation of the components, etc., which can change the values of the components parameters. As a result of these destabilizing actions, after some time interval, the actual characteristics of the filters may differ from the characteristics calculated during the design.

Because of this, in order to evaluate the stability of the filter characteristics as a result of the action of destabilizing factors, but also to evaluate the possible tolerances of the nominal components, the sensitivity of the corresponding characteristics is carried out in the field of the multidimensional space of the parameters of the filter components.

Usually, to assess the action of the X parameter on the Y characteristic of the filter, the logarithmic or relative sensitivity of the Y characteristic to the change of the X parameter value is most frequently used, which is denoted $\boldsymbol{S_X^Y}$, introduced by Bode [3] and calculated from the relation:

$$S_X^Y = \frac{\partial\,(lnY)}{\partial\,(lnX)} = \frac{X}{Y} \cdot \frac{\partial Y}{\partial X} \qquad (1)$$

The S_X^Y value, calculated in this way, represents a complex quantity. In the theory of electronic circuits it is demonstrated that the real part of the sensitivity function $Re\,S_x^{T(x,p)}$ of the transfer function of the electronic circuit (of the filter, in the given case) describes the sensitivity of the Amplitude - Frequency Characteristic (AFC) of the filter, and the imaginary part $Im\,S_x^{T(x,p)}$ describes the sensitivity of the Phase-Frequency Characteristic (PFC) of the filter.

II. AFC SENSITIVITY ON THE REAL FILTER

The AFC sensitivity of the filter to the change in the value of an element of the electronic circuit of the filter indicates the value of the change in the signal level in the pass and hold bands of the filter when the value of the indicated element changes and is called the AFC sensitivity to this element, but taking into account the changes in the values of all In the elements of the electronic circuit, the summary sensitivity of the AFC of the filter is obtained, which can be calculated according to the relationship:

$$S_\Sigma = \sum_{i=1}^{n} Re\,[\frac{X_i}{T(p,X_i)} \cdot \frac{\partial T(p,X_i)}{\partial X_i}] \qquad (2)$$

In the project, the relations for the sensitivity of the Amplitude - Frequency Characteristic to each element of the electronic circuit of the filter in figure 1, which represents the scheme of a low-pass filter (LPF), are obtained.

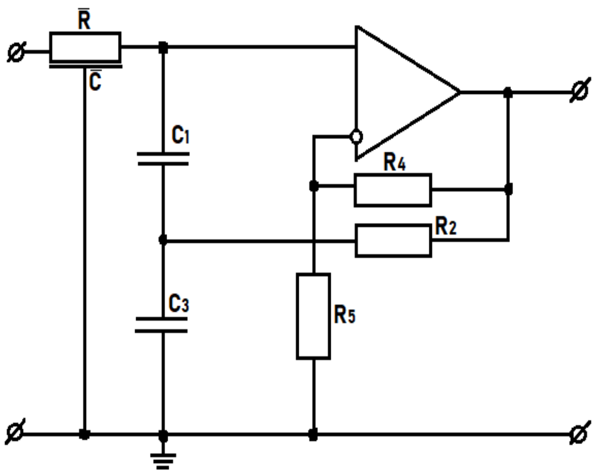

Figure 1. Analyzed circuit

The transfer function of the filter in figure 1 is described with the following relation [4]:

$$T(\theta) = \frac{1+(1+n)\alpha\theta^2}{[1+(1+n)\theta^2]ch\theta+\beta\theta(\alpha n\theta^2-\gamma)sh\theta)} \qquad (3)$$

where $\theta = \sqrt{p\overline{RC}}$; $\quad p = j\omega$; $\quad n = \frac{C_3}{C_1}$;
$\beta = \frac{C_1}{C}$; $\gamma = \frac{R_4}{R_5}$; $\alpha = \frac{C_1 R_2}{CR}$.

To calculate the sensitivity of the transfer function of the filter with respect to the parameter $\boldsymbol{\alpha}$, which will be noted by $S_\alpha^{T(\Theta,\alpha)}$ it is necessary to calculate the first partial derivative of the transfer function of the filter with respect to the parameter $\boldsymbol{\alpha}$ which has the following form:

$$\frac{\partial T(\theta,\alpha)}{\partial \alpha} = \frac{\beta\theta^3[\gamma(1+n)+n]sh\theta}{\{[1+(1+n)\alpha\theta^2]ch\theta+\beta\theta(\alpha n\theta^2-\gamma)sh\theta\}^2} \qquad (4)$$

The result we obtain:

$$S_{\alpha}^{T(\theta,\alpha)} = \frac{\partial T(\theta,\alpha)}{\partial \alpha} \cdot \frac{\alpha}{T(\theta,\alpha)} = \frac{A(\theta)}{B(\theta)} \quad (5)$$

where

$$A(\Theta) = -\alpha\beta\Theta^3[\gamma(1+n)+n]sh\Theta \quad (6)$$

$$B(\Theta) = [1+(1+n)\alpha\Theta^2]^2 ch\Theta + \beta\Theta(\alpha n\Theta^2 - \gamma)sh\Theta \quad (7)$$

Therefore the AFC sensitivity of the analyzed filter$S_{\alpha}^{T(\theta,\alpha)}$ obtain the form:

$$S_{\alpha}^{|T(j\omega,\alpha)|} = \frac{ReA*ReB+ImA*ImB}{(ReB)^2+(ImB)^2} \quad (8)$$

where:

$$ReA = -alx\beta[\gamma\left(\frac{1}{n}+1\right)+1] \quad (9)$$

$$ImA = blx\beta[\gamma\left(\frac{1}{n}+1\right)+1] \quad (10)$$

$$ReB = c - \beta x(yb - la) - k[kc + 2d + \beta x(lb - ya)] \quad (11)$$

$$ImB = d + \beta x(lb - ya) + k[2c - kd - \beta x(yb - la)] \quad (12)$$

and

$$a = shx\cos x + shx\, sinx; b = shx\, cosx - chxsinx;$$

$$l = 2\alpha nx^2; k = 2(1+n)\alpha x^2; c = chxcosx;$$

$$d = chxsinx; x = \sqrt{0{,}5\omega RC}.$$

In analogical mode, the expressions of the Amplitude - Frequency Characteristic sensitivity of the analyzed filter to changing the values of its other elements were calculated. The corresponding sensitivities are calculated from the relations:

— on the β parameter, which indicates the stability of capacitor capacities C_1 and capacity of $\boldsymbol{RC}$ – element and more importantly indicates the stability of the relationship of these capacities

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$$S_{\beta}^{T(\theta,\beta)} = \frac{K}{Lch\theta+K}; \quad (14)$$

where:

$$K = \beta\theta(\alpha n\theta^2 - \gamma)sh\theta \quad (15)$$

$$L = 1 + (1+n)\alpha\theta^2; \quad (16)$$

-on the $\boldsymbol{n}$ parameter, which indicates the stability of the capacities of capacitors C1 and C3 and their ratio

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$$S_{n}^{T(\theta,n)} = \frac{\alpha\beta n\Theta^3(1+\gamma+\alpha\Theta^2)sh\Theta}{L(L\, ch\Theta + K)} \quad (17)$$

- on the variable $\boldsymbol{\theta}$ (the sensitivity of the transfer function of the filter to the change in the value of the time constant of the RC-element with distributed parameters

$$S_{\Theta}^{T(\Theta)} = \frac{Z+Q}{L(Lch\Theta+K)} \quad (18)$$

where $Z = \Theta[2k\alpha(1+n)\Theta - L]$ (19)

$$Q = [Kcoth\Theta + (L + 3\alpha\beta n\Theta^2 - \gamma\beta)]sh\Theta \,. \quad (20)$$

- on the γ parameter :

$$S_{\gamma}^{T(\theta,\gamma)} = \frac{\gamma\beta\Theta sct\Theta}{Lch\Theta+K} \quad (21)$$

Where are used the notes from equation (3).

III. CONCLUSIONS

The calculation of the Amplitude - Frequency Characteristic sensitivity of the filter according to expressions 8, 14, 17, 18,21 was carried out by numerical methods on the computer.

According to the obtained values of Amplitude - Frequency Characteristic sensitivities to the change in the values of the components of the electronic circuit, the deviation (difference) of the Amplitude - Frequency Characteristic of the filter compared to the calculated Amplitude - Frequency Characteristic can be evaluated.

To generate the results obtained from the analysis of the concrete electronic circuit with real values of all its components, namely: the $\boldsymbol{RC}$ element, capacitors C_1 and C_3 and resistors R_2, R_4 and R_5 the results obtained in a frequency band, described by a more generalized frequency $\boldsymbol{\lambda = pRC}$, which allows the results obtained to be extended to circuits with various values of the components.Some results of dependency calculations $S_{x_i}^{|T(j\omega)|}$ when creating the second-rank filter are indicated in figure 2. From the dependencies in the figure it can be seen that the Amplitude - Frequency Characteristic sensitivity is directly proportional to the value of the parameter **α** and inversely proportional to the parameter value **γ**.

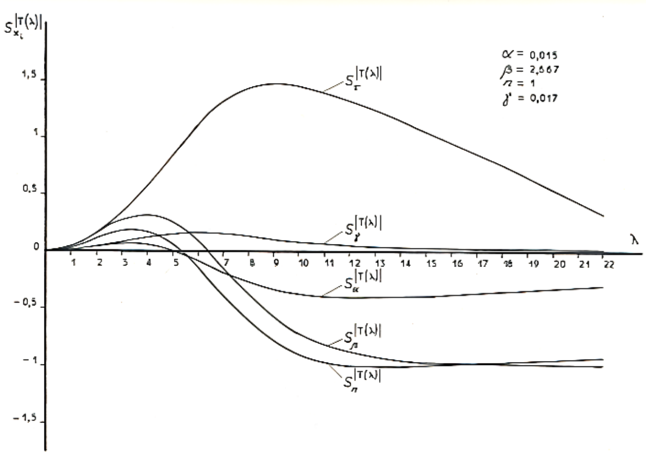

Figure 2. Amplitude - Frequency Characteristic sensitivity of the analyzed Low Pass Filter

At the same time, the maximum values of the sensitivity functions on each element of the Amplitude - Frequency Characteristic are found near the cutoff frequency of the Low Pass Filter.

The analysis of the graphs in figure 2 allows us to conclude that the Amplitude - Frequency Characteristic sensitivities values to the Low Pass Filter elements of the second degree are in the band (-1, +1.47). It should be noted that the maximum sensitivity of the analyzed Low Pass Filter (i.e. +1.47) manifests itself precisely at changes in the nominal values of the ***RC*** element.

The realization of the poles of the transfer function of the analyzed Low Pass Filter with quality factors $Q_p>1$ always results in the increase of the values of the sensitivities of the Amplitude - Frequency Characteristic towards each element and the sum sensitivity of the Amplitude - Frequency Characteristic. Theoretical and practical research has shown that, for example, if it is necessary to increase the Amplitude - Frequency Characteristic level for the analyzed second-order filters by +3 dB, this results in an increase in the level of Amplitude - Frequency Characteristic sensitivities to each element up to values of +2.3. The maximum summary sensitivity of the Amplitude - Frequency Characteristic reaches the value of +4.69 for the second degree Low Pass Filter analyzed.

Based on the obtained results, it can be concluded that the second-degree Low Pass Filter made on the basis of the electronic circuit in figure 1 has small sensitivities to the change of nominal components that do not exceed the value of 1 on average, up to 1.47 for a single component of the Low Pass Filter. Therefore, on the basis of the electronic circuit in figure 1, Low Pass Filters with a fairly high stability of the Amplitude - Frequency Characteristic can be realized, which will result in a prescribed operation of the transmission and reception channels of the telecommunications and information networks.

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