

## MAXIMUM VALUES OF FROZEN SOIL'S DEPTH AND HOAR FROST'S THICKNESS ESTIMATED ON THE BASIS OF EXTREME VALUES THEORY

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**Abstract.** Extreme climatic phenomena present risk factors for agriculture, health, constructions, etc. and are studied profoundly these past years using extreme value theory. Several relations that describe positive extreme values' probability Generalized Extreme Value and Gumbel distribution are presented in the article. As an example, we show the maps of characteristic and reference values of the maximum depth of the frozen soil and thickness of hoar-frost with a probability of exceeding per year equal to 0,02, which is equivalent to the mean return interval of 50 years. The obtained results could serve as a base for elaboration of national annexes in constructions.

### Introduction

Extreme weather events are risk factors for the environment, agriculture, health, road, rail and air transport, construction, etc. These phenomena cause human and material damage and are intensively studied in recent decades using extreme value theory. Global warming has become a reality. There is an intensification and increased frequency of extreme weather events' occurrence.

As a result, there is a need to study the temporal and spatial distribution of climatic risk factors, including those in the construction industry. As an example, we present maps of the characteristic (reference) values of the depth of soil frost and hoar frost's thickness with certain return periods.

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Frozen terrain can cause cracks in the building foundation due to increased volume in the process of water transformation into ice (STAS 6054, 1977, CII 20.13330, 2016, ДБН В.1.2-2, 2006 normative documents). Durability depends on the depth of the foundation of the building is determined by taking into account the depth of the frost, the soil-forming rock, the groundwater level, the constructive minimum height of the foundation and technological conditions of the loads exerted on the foundation of the building. The foundation depth can not be less than the maximum freezing depth during the lifetime of the building. When designing roads, the total thickness of the asphalt and the floor must not be less than the maximum freezing depth during exploitation period.

Hoar frost is formed by white, crumbly, snow-like ice crystals, mainly on branches, conductors, corners and edges of objects, usually in frosty weather, in the presence of mist and weak wind.

Hoar frost loads should be taken into account for power lines and air communications, electrified transport contact networks, antenna devices as well as ventilated facades of buildings, metal balconies (CII 20.13330, 2016, ДБН В.1.2-26 2006, NTE 003/04/00, 2004 normative documents).

The freezing depth of the soil is measured in cm, and the thickness of the hoar frost - in mm.

### 3. Materials and research methods

As initial materials, the annual maximums of frozen soil's thickness and hoar-frost thickness were used for the period of 1961-2016, recorded at the meteorological stations of the State Hydro-meteorological Service. The development of digital maps was performed in ArcGIS 10 by interpolation of the reference values (characteristic) with the mean return period of 50 years, calculated for each meteorological station.

The distribution of maximum climatic values, especially the annual ones, is expressed by the Generalized Extreme Value Distribution ([www.mathwave.com](http://www.mathwave.com)):

$$G(x; \mu, \sigma, \xi) = \exp \{-[1 + \xi(x - \mu)/\sigma]\}^{-1/\xi}, \quad (1),$$

where  $-\infty < \mu < \infty$ ,  $\sigma > 0$  and  $-\infty < \xi < \infty$  are the location, scale, and shape parameters. Depending on the value  $\xi$ , the expression (1) defines three types of distribution: Weibull ( $\xi < 0$ ), Gumbel ( $\xi = 0$ ) and Fréchet ( $\xi > 0$ ).

Majority of European countries use Gumbel distribution (Engineering Statistics Handbook) for evaluation of climatic extremes values in construction

field, expressed by probability density function (PDF) and cumulative distribution function (CDF):

$$f(x) = (1/\sigma) \exp(-z - \exp(-z)) \quad (2),$$

$$F(x) = \exp(-\exp(-z)) \quad (3),$$

where  $z = (x - \mu)/\sigma$ ,  $\mu$ , și  $\sigma$  – location and scale (distribution's parameters),  $f(x) = dF(x)/dx$ .

Distribution's parameters can be expressed by mean  $x_{med}$  and standard deviation  $\sigma_1$  of the sample:  $\mu = x_{med} - \gamma \sigma$ , where  $\gamma \approx 0.5772$  – Euler-Mascheroni constant,  $\sigma = (\sqrt{6/\pi}) \sigma_1$ . Consequently,  $\mu = x_{med} - 0.45 \sigma_1$  and  $\sigma = 0.7797 \sigma_1$ . 4).

Quintile  $x(p)$  is the inverse function of the cumulative distribution function  $F(x)$ .

In Gumbel distribution for maximums quintile is expressed by the relation:

$$x(p) = \mu - \sigma \ln(-\ln(p)) \quad (5)$$

$$\text{Consequently, } x(p) = x_{med} - \{0.45 + 0.7797 * \ln[\ln(1/p)]\} \sigma_1 \quad (6)$$

The reference (characteristic) value to be exceeded in one year with the probability  $p$  is equal to  $x(1-p) = x_{med} - \{0.45 + 0.7797 * \ln[\ln(1/(1-p))]\} \sigma_1$  (7)

Reference value to be exceeded in one year with probability  $p = 0,02$  (average return period IMR=50 years) is equal to  $x(0,98) = x_{med} + 2.5923 \sigma_1$  (8)

Using relation (8) is the simplest procedure of evaluation of reference values of extremes with average return period of once per fifty years. Relation (7) can be used for calculating the values with other return periods.

There is an extensive range of software which can be used for evaluation of extreme values' parameters distribution: Statgraphics Centurion, Matlab, Statistica, EasyFit, etc., or modules generating graphs and tables elaborated in R language.

#### 4. Obtained results

As initial data, we had used the maximum annual values of the depth of soil frost in cm, recorded at 17 meteorological stations of the State Hydrometeorological Service between 1961 and 2016 (56 years), with some exceptions (Bălți - 30 years, Bravicea - 55 stations Cahul - 55 years, Camenca - 54 years, Ceadir-Lunga - 55 years, Comrat - 52 years, Cornești - 50 years, Dubăsari - 54 years, Leova - 54 years, Voda - 55 years, Tiraspol - 54 years).

The maximum annual values are obtained from the maximum values for 5 days or decades for the November-April period.

The mean value and standard deviation of the frozen soil's depth and the Href reference value corresponding to the 50-year return period according to the equation (8) were calculated for each meteorological station. Href values were spatially interpolated using Spline method (Radial Basic Functions, Minimum Curvature) in ArcGIS. The map of the spatial distribution of frozen soil's depth reference values with the 50-year average return period is shown in Fig. 1.

Href field varies within 60-110 cm in the field. The corresponding values are indicated on the Href isolines. Average values are observed in the Central-Western part of the country. Minimal values are characteristic of the South and South East of the country. The highest values are observed in the North and Central-East of the territory. Table 1 presents the reference values extracted from the digital map for 57 cities and municipalities in the Republic of Moldova.

The normative value (9-12) of hoar frost's linear load for cross-sectional elements up to 70 mm in diameter (wires, cables, bars, pillars, etc.),  $i$  N/m must be determined by the formula:

$$i = \pi b k_1 \mu_1 (d + b k_1 \mu_1) \rho g 10^{-3} \quad (9)$$

The normative value of the surface ice load (9-12),  $i'$  Pa, for the remaining structural elements should be determined by the formula:

$$i' = b k_1 \mu_2 \rho g \quad (10),$$

where  $b$  is the normalized thickness of the hoar frost, mm (exceeded in average once in 5 years), for circular cross-sectional elements with 10 mm diameter, located at a height of 10 m above the ground.  $k$  is the- the coefficient which takes into account hoar frost's thickness change depending on the height. For  $h = 10$  m,  $k = 1.0$ ,  $\mu_1$  is the coefficient which takes into account the change in the thickness of the hoar frost depending on the diameter of the element. For  $d = 10$  mm,  $\mu_1 = 1.0$ ,  $\mu_2$  is the coefficient which takes into account the ratio of the surface area of the element subjected to hoar frost accumulation to the entire surface of the element and which equals to 0.6 for objects with small cross-sections and for other objects it is based on the special investigation data.  $\rho$  is the ice density, 0,9 g/cm and  $g$ , m/s, is the free fall acceleration.

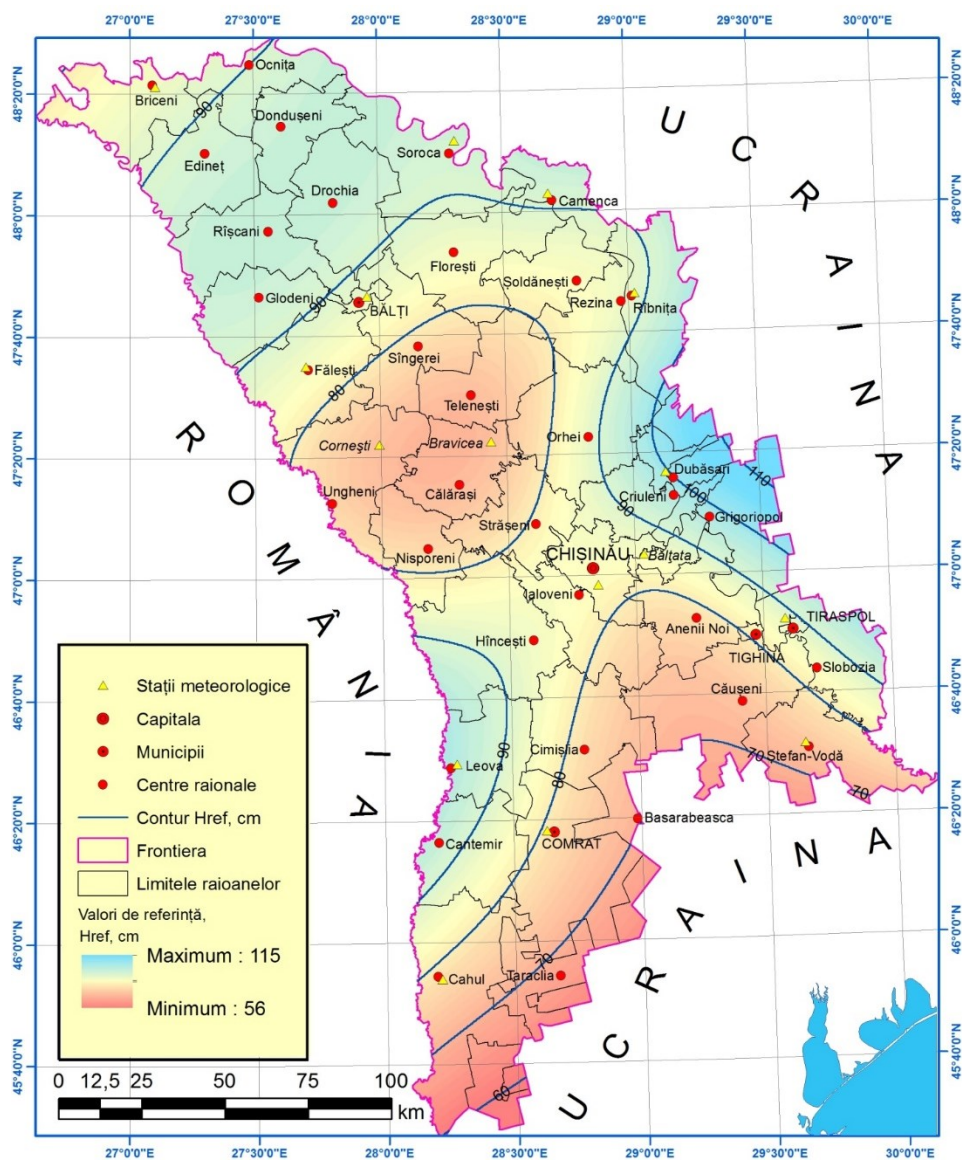


Fig.1. Spatial distribution map of the reference values of the frozen soil's depth with mean return period equal to 50 years

**Table 1.** The reference values of the frozen soil's depth for 58 localities in the Republic of Moldova with an average return period of 50 years (IMR = 50)

Nr.	Locality	Region/municipality	Href, cm
1	Anenii Noi	Anenii Noi	79
2	Bălțata*	Mun. Chișinău	85
3	Bălți	Mun. Bălți	87
4	Bălți*	Mun. Bălți	87
5	Basarabasca	Basarabasca	71
6	Bravicea*	Călărași	73
7	Briceni	Briceni	86
8	Briceni*	Briceni	87
9	Cahul	Cahul	79
10	Cahul*	Cahul	79
11	Călărași	Călărași	72
12	Camenca	Camenca	90
13	Camenca*	Camenca	91
14	Cantemir	Cantemir	93
15	Căușeni	Căușeni	74
16	Ceadr-Lunga*	Ceadr-Lunga	68
17	Chișinău	Mun. Chișinău	84
18	Chișinău*	Mun. Chișinău	84
19	Cimișlia	Cimișlia	78
20	Comrat	Mun. Comrat	79
21	Comrat*	Mun. Comrat	80
22	Cornești*	Ungheni	73
23	Criuleni	Criuleni	97
24	Dondușeni	Dondușeni	92
25	Drochia	Drochia	93
26	Dubăsari	Dubăsari	99
27	Dubăsari*	Dubăsari	100
28	Edineț	Edineț	91
29	Fălești	Fălești	85
30	Fălești*	Fălești	86
31	Florești	Florești	86
32	Glodeni	Glodeni	92

Nr.	Locality	Region/municipality	Href, cm
33	Grigoriopol	Grigoriopol	97
34	Hîncești	Hîncești	86
35	Ialoveni	Ialoveni	84
36	Leova	Leova	96
37	Leova*	Leova	96
38	Nisporeni	Nisporeni	77
39	Ocnîța	Ocnîța	90
40	Orhei	Orhei	88
41	Rezina	Rezina	87
42	Rîbnița	Rîbnița	88
43	Rîbnița*	Rîbnița	89
44	Rîscani	Rîscani	93
45	Sîngerei	Sîngerei	78
46	Slobozia	Slobozia	84
47	Șodanești	Șodanești	84
48	Soroca	Soroca	91
49	Soroca*	Soroca	92
50	Ștefan-Vodă	Ștefan-Vodă	74
51	Ștefan-Vodă*	Ștefan-Vodă	75
52	Strășeni	Strășeni	81
53	Taraclia	Taraclia	68
54	Telenești	Telenești	73
55	Tighina	Mun. Tighina	82
56	Tiraspol	Mun. Tiraspol	87
57	Tiraspol*	Tiraspol	88
58	Ungheni	Ungheni	77

Note: \* –meteorological station

Spatial distribution map of the reference values of the thickness of the hoar frost  $b$ , mm, with an average return period of 50 years is shown in Fig. 2.  $b$ , varies within the range of 8.4-29.5. The corresponding values are indicated on the isolines  $b$ . The highest values are observed in the North and West (in central and southern parts) of the territory.

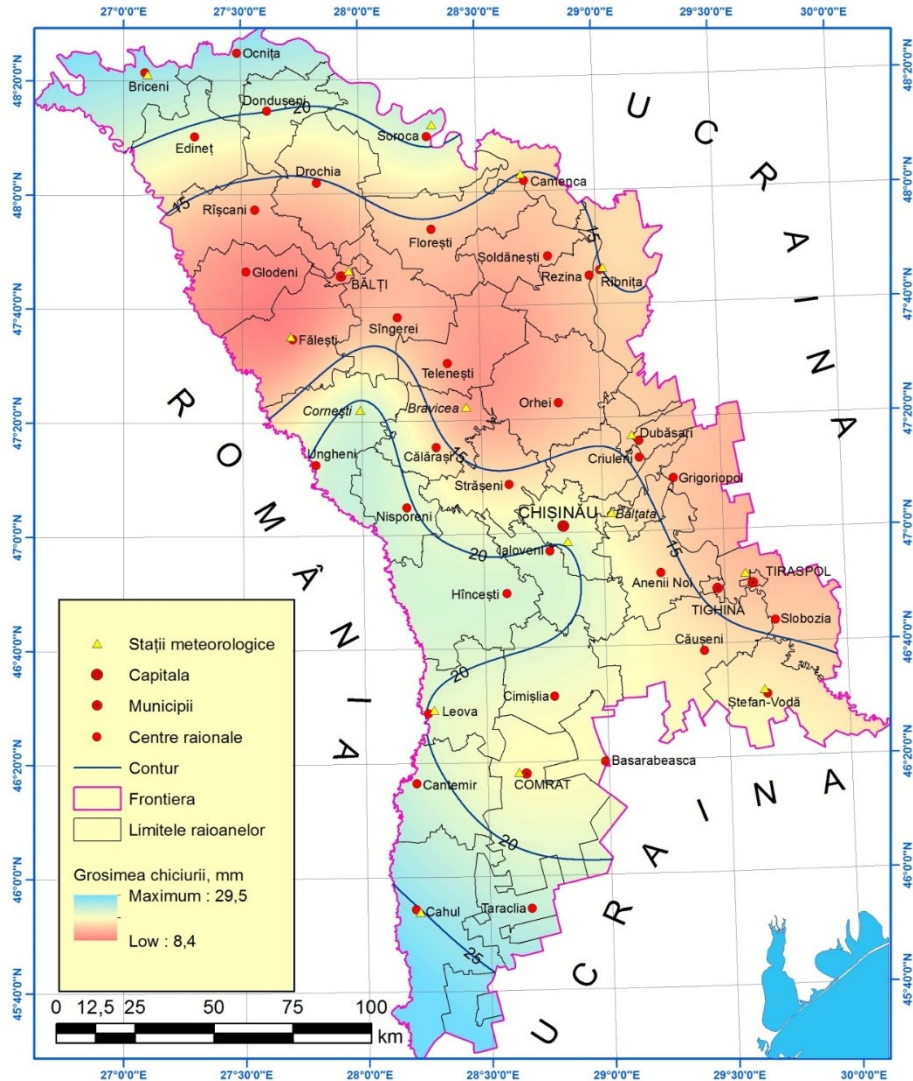


Fig.2. Spatial distribution map of the reference values of the hoar frost's thickness with mean return period equal to 5 years

The reference values of the thickness of the hoar frost may be used to determine the linear load of hoar frost on high voltage power lines and electrical transmission lines using equation (9).

The equation (10) can be used for hoar- frost's surface load.



**Table 2.** The reference values of the hoar frost's thickness  $b$ , mm, for for 58 localities from Republic of Moldova (mean return period is equal 5 years)

Nr	Locality	Region/municipality	$b$ , mm
1	Anenii Noi	Anenii Noi	16.1
2	Bălțata*	Mun. Chișinău	16.7
3	Bălți	Mun. Bălți	11.1
4	Bălți*	Mun. Bălți	11.5
5	Basarabeasca	Basarabeasca	18.1
6	Bravicea*	Călărași	11.4
7	Briceni	Briceni	24.5
8	Briceni*	Briceni	24.3
9	Cahul	Cahul	23.2
10	Cahul*	Cahul	25.1
11	Călărași	Călărași	16.0
12	Camenca	Camenca	14.6
13	Camenca*	Camenca	14.9
14	Cantemir	Cantemir	21.1
15	Căușeni	Căușeni	16.3
16	Ceadr-Lunga*	Ceadr-Lunga	20.1
17	Chișinău	Mun. Chișinău	19.1
18	Chișinău*	Mun. Chișinău	19.8
19	Cimișlia	Cimișlia	18.5
20	Comrat	Mun. Comrat	17.8
21	Comrat*	Mun. Comrat	17.9
22	Cornești*	Ungheni	20.8
23	Criuleni	Criuleni	14.7
24	Dondușeni	Dondușeni	20.0
25	Drochia	Drochia	14.7
26	Dubăsari	Dubăsari	14.6
27	Dubăsari*	Dubăsari	14.6
28	Edineț	Edineț	24.3
29	Fălești	Fălești	9.5

Nr.	Locality	Region/municipality	<i>b</i> , mm
30	Fălești*	Fălești	9.3
31	Florești	Florești	14.4
32	Glodeni	Glodeni	9.6
33	Grigoriopol	Grigoriopol	14.1
34	Hîncești	Hîncești	20.7
35	Ialoveni	Ialoveni	29.1
36	Leova	Leova	20.0
37	Leova*	Leova	19.8
38	Nisporeni	Nisporeni	20.3
39	Ocnîța	Ocnîța	24.3
40	Orhei	Orhei	13.0
41	Rezina	Rezina	14.6
42	Rîbnița	Rîbnița	15.0
43	Rîbnița*	Rîbnița	15.0
44	Rîscani	Rîscani	12.8
45	Sîngerei	Sîngerei	13.3
46	Slobozia	Slobozia	14.4
47	Șodanești	Șodanești	13.3
48	Soroca	Soroca	20.6
49	Soroca*	Soroca	21.5
50	Ștefan-Vodă	Ștefan-Vodă	16.9
51	Ștefan-Vodă*	Ștefan-Vodă	16.8
52	Strășeni	Strășeni	16.0
53	Taraclia	Taraclia	22.0
54	Telenești	Telenești	11.9
55	Tighina	Mun. Tighina	14.3
56	Tiraspol	Mun. Tiraspol	13.3
57	Tiraspol*	Tiraspol	13.2
58	Ungheni	Ungheni	20.5

Note: \* –meteorologic station

### Conclusions

The analysis of the annual maximum values of frozen soil's thickness and hoar frost thickness in the Gumbel distribution allowed to obtain the reference values with a certain average return time at each meteorological station.

Reference values can be used to elaborate digital maps, which can be used to obtain the corresponding values for certain localities.

The maps and tables presented in the work can be used for the elaboration of normative documents in constructions for the Republic of Moldova.

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