Some particular cases for inverse operations in the class of preradicals in modules

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In [1], [2], [3] four new operations are introduced and studied in the class of preradicals $\mathbb{PR}$ in modules, namely, the inverse operations of the product and of the coproduct with respect to meet and to join. They are defined as follows:

(1) the *left quotient with respect to join* $r \,{}^{\vee}\!/_{\cdot}\, s = \vee \{r_\alpha \in \mathbb{PR} \mid r_\alpha \cdot s \leq r\}$, which exists $\forall\, r, s \in \mathbb{PR}$;

(2) the *left coquotient with respect to meet* $r \,{}^{\wedge}\!/_{\#}\, s = \wedge \{r_\alpha \in \mathbb{PR} \mid r_\alpha \# s \geq r\}$, which exists $\forall\, r, s \in \mathbb{PR}$;

(3) the *left quotient with respect to meet* $r \,{}^{\wedge}\!/_{\cdot}\, s = \wedge \{r_\alpha \in \mathbb{PR} \mid r_\alpha \cdot s \geq r\}$, which exists $\forall\, r, s \in \mathbb{PR}, r \leq s$;

(4) the *left coquotient with respect to join* $r \,{}^{\vee}\!/_{\#}\, s = \vee \{r_\alpha \in \mathbb{PR} \mid r_\alpha \# s \leq r\}$, which exists $\forall\, r, s \in \mathbb{PR}, r \geq s$.

The similar questions are discussed in [4; 5; 6].

In this communication some important particular cases of these operations are considered. Namely, for each of formulated operation we indicate a particular case, which coincides with a well known operator in $\mathbb{PR}$. Moreover, some properties of these operators are shown [1; 2; 3; 7; 8].

For any preradical $r \in \mathbb{PR}$, these particular cases are:

(1) $0 \,{}^{\vee}\!/_{\cdot}\, r = \vee \{r_\alpha \in \mathbb{PR} \mid r_\alpha \cdot r = 0\} = a(r)$ is the *annihilator* of r;

(2) $1 \,{}^{\wedge}\!/_{\#}\, r = \wedge \{r_\alpha \in \mathbb{PR} \mid r_\alpha \# r = 1\} = t(r)$ is the *totalizer* of r;

(3) $r \,{}^{\wedge}\!/_{\cdot}\, r = \wedge \{r_\alpha \in \mathbb{PR} \mid r_\alpha \cdot r = r\} = e(r)$ is the *equalizer* of r;

(4) $r \,{}^{\vee}\!/_{\#}\, r = \vee \{r_\alpha \in \mathbb{PR} \mid r_\alpha \# r = r\} = c(r)$ is the *co-equalizer* of r.

The annihilator of preradical r possesses the following properties $\forall\, r \in \mathbb{PR}$:

(1) $a(r) \cdot r = 0$;

(2) $a(r)$ is a radical;

(3) $a(s) \leq r \, {}^{\vee}\!/_{\cdot} \, s, \ \forall s \in \mathbb{PR}$;

(4) $r^{\perp} \leq a(r) \leq r^{*}$, where $r^{\perp}$ is pseudocomplement and r^{*} is supplement of preradical r.

The totalizer of preradical r possesses the following properties $\forall r \in \mathbb{PR}$:

(1) $t(r) \# r = 1$;

(3) $t(r)$ is a Jansian pretorsion;

(3) $t(s) \geq r \, {}^{\wedge}\!/_{\#} \, s, \ \forall s \in \mathbb{PR}$;

(4) $r^{\perp} \leq t(r) \leq r^{*}$.

The equalizer of preradical r possesses the following properties $\forall r \in \mathbb{PR}$:

(1) $e(r) \cdot r = r$;

(2) $e(r)$ is an idempotent preradical;

(3) r is an idempotent preradical $\Leftrightarrow$ $e(r) = r$;

(4) $r \leq r \, {}^{\wedge}\!/_{\cdot} \, s \leq e(r), \ \forall s \in \mathbb{PR}, \ s \geq r$;

(5) $e(r) \cdot (r \, {}^{\wedge}\!/_{\cdot} \, s) = r \, {}^{\wedge}\!/_{\cdot} \, s, \ \forall s \in \mathbb{PR}, \ s \geq r$;

(6) $(r \, {}^{\wedge}\!/_{\cdot} \, s) \cdot e(s) = r \, {}^{\wedge}\!/_{\cdot} \, s, \ \forall s \in \mathbb{PR}, \ s \geq r$;

(7) $(r \, {}^{\wedge}\!/_{\cdot} \, s) \, {}^{\wedge}\!/_{\cdot} \, e(s) = r \, {}^{\wedge}\!/_{\cdot} \, s, \ \forall s \in \mathbb{PR}, \ s \geq r$.

The co-equalizer of preradical r possesses the following properties $\forall r \in \mathbb{PR}$:

(1) $c(r) \# r = r$;

(2) $c(r)$ is a radical;

(3) r is a radical $\Leftrightarrow$ $c(r) = r$;

(4) $c(r) \leq r \, {}^{\vee}\!/_{\#} \, s \leq r, \ \forall s \in \mathbb{PR}, \ s \leq r$;

(5) $c(r) \# (r \, {}^{\vee}\!/_{\#} \, s) = r \, {}^{\vee}\!/_{\#} \, s, \ \forall s \in \mathbb{PR}, \ s \leq r$;

(6) $(r \, {}^{\vee}\!/_{\#} \, s) \# c(s) = r \, {}^{\vee}\!/_{\#} \, s, \ \forall s \in \mathbb{PR}, \ s \leq r$;

(7) $(r \, {}^{\vee}\!/_{\#} \, s) \, {}^{\vee}\!/_{\#} \, c(s) = r \, {}^{\vee}\!/_{\#} \, s, \ \forall s \in \mathbb{PR}, \ s \leq r$.

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