On topological endomorphism rings with no more than two non-trivial closed ideals

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Let $\mathcal{L}$ be the class of locally compact abelian groups. For $X \in \mathcal{L}$, we denote by $t(X)$ the torsion subgroup of X and by $E(X)$ the ring of continuous endomorphisms of X, taken with the compact-open topology. If X is topologically torsion, then $S(X)$ stands for the set of primes p such that the corresponding topological p-primary component of X is non-zero. Given a positive integer n, we set $nX = \{nx \mid x \in X\}$ and $X[n] = \{x \in X \mid nx = 0\}$.

Theorem 1. *Let n be a positive integer, and let X be a group in $\mathcal{L}$ such that $\overline{nX}$ is densely divisible and $t(X) = X[n]$. If $E(X)$ has no more than two non-trivial closed ideals, then X is either topologically torsion or topologically isomorphic with the topological direct product of a topologically torsion group by a group of the form $\mathbb{R}^d$, $\mathbb{Q}^{(\mu)}$, or $(\mathbb{Q}^*)^\mu$, where d is a positive integer and μ is a non-zero cardinal*

Theorem 2. *Let X be a group in $\mathcal{L}$ such that $E(X)$ has no more than two non-trivial closed ideals. If X is topologically torsion, then $|S(X)| \leq 2$. If X is topologically isomorphic with a group of the form $S \times T$, where T is topologically torsion and S is either $\mathbb{R}^d$ for some positive integer d, or $\mathbb{Q}^{(\mu)}$ or $(\mathbb{Q}^*)^\mu$ for some non-zero cardinal number μ, then $|S(X)| \leq 1$.*