Synthesis of the minimum variance control law for the linear time variant processes

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Above the linear process can act the disturbance signals, so kind of processes can be described by the parametrical models, where the most used take part from the ARMAX class (Auto-Regressive Moving Average with eXogenous control). The general model of the class is the ARMAX model $[na, nb, bc, nk]$, which in fact represents that the output signal is obtained as a result of the superposition between a useful signal obtained by filtering the input signal and a parasitic signal obtained by filtering the white noise :

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{C(q^{-1})}{A(q^{-1})}e(k) = y^u(k) + y^e(k).$$

where $y(k)$ is the output of the noisy system, $u(k)$ - control signal, $e(k)$ is a sequence of independent normal variables with zero mean value and variance one (white noise) and the polynomials $A(q^-1), B(q^-1), C(q^-1)$ are

$$A(q^{-1}) = 1 + a_1 q^{-1} + ... + a_{na} q^{-na},$$

$$B(q^{-1}) = b_1 q^{-1} + ... + b_{nb} q^{-nb},$$

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{nc} q^{-nc},$$

where q^{-1} is backshift operator.

To determine the optimal control, it is used the performance criterion that provides the minimum variance value of the output The purpose of the minimum variance control is to determine the control signal $u(k)$ in such a way that the loss function

$$J = E\{y^2(k)\}.$$

is as small as possible and the control law that ensures the minimization of the given criteria is called the minimum variance control.

The component $y^e(k)$ represents the influence of the environment on the process and it is characterized by the stochastic disturbance signals and is given by

$$y^e(k) = \frac{C(q^{-1})}{A(q^{-1})} e(k).$$

If the output at the k and $k-1$ tact are observed, then the output at the m tact is

$$y(k+m) = \frac{C(q^{-1})}{A(q^{-1})} e(k+m) = F(q^{-1}) e(k+m) + \frac{q^{-m} G(q^{-1})}{A(q^{-1})} e(k+m).$$

To obtain the polynomials $F(q^{-1})$ and $G(q^{-1})$ it is necessary to be solved the diophantine equation:

$$C(q^{-1}) = A(q^{-1}) F(q^{-1}) + q^{-m} G(q^{-1}).$$

In this way, the control law is represented by the following equation:

$$u(k) = -\frac{G(q^{-1})}{A(q^{-1}) B(q^{-1})}.$$

The output of the system under the control of the minimum variance in the stationary regime regime is:

$$y(k) = F(q^{-1}) e(k) = e(k) + f_1 e(k-1) + ... + f_d e(k-d).$$

And the variance of the estimator error can be determinate by the

$$\sigma_y^2 = (1 + f_1^2 + ... + f_d^2) \sigma_e^2.$$

It is given the thermic process of temperature variation in a oven. The mathematical model that approximates the temperature variation in a oven was obtained based on the MATLAB software and it is

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + \frac{C(q^{-1})}{A(q^{-1})} e(k) =$$
$$= \frac{0.0000021 q^{-1} + 0.000002158 q^{-2}}{1 - 1.991 q^{-1} + 0.991 q^{-2}} u(k) + \frac{1 - 1.229 q^{-1} + 0.2402 q^{-2}}{1 - 1.991 q^{-1} + 0.991 q^{-2}} e(k).$$

The diofantic equation for solving the polimomials $F(q^{-1})$ and $G(q^{-1})$ is

$$(1 - 1.229 q^{-1} + 0.2402 q^{-2}) = (1 - 1.991 q^{-1} + 0.991 q^{-2})(1 + f_1 q^{-1} + f_2 q^{-2}) +$$
$$+ q^{-3}(g_0 + g_1 q^{-1}).$$

The control law can be presented in the following way:

$$u(k) = -\frac{(0.77058 - 0.7594 q^{-1})}{(0.0000021 q^{-1} + 0.000002158 q^{-2})(1 + 0.762 q^{-1} + 0.766 q^{-2})}.$$

The variance of the estimator error is $\sigma_y = 2.1674$.

Bibliography

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