is a minimal rational basis of the $GL(2,\mathbb{R})$-comitants for the system (1) of differential equations of the fifth degree on $M = \{ a \in A \mid R_1 \not\equiv 0 \ (K_{101} \not\equiv 0)\}$.

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Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations.* Manchester University Press, 1988.

[2] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants.* Noordhoff, Groningen, 1964.

[3] Iu. Calin, *On rational bases of $GL(2,\mathbb{R})$-comitants of planar polynomial systems of differential equations.* Buletinul Academiei de Stiinte a Republicii Moldova. Matematica, 2003, nr. 2 (42), pp. 69-86.

The sufficient center conditions for a class of bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree

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Let us consider the system of differential equations with nonlinearities of the fourth degree

$$\frac{dx}{dt} = P_1(x,y) + P_4(x,y), \quad \frac{dy}{dt} = Q_1(x,y) + Q_4(x,y), \tag{1}$$

where $P_i(x,y)$ and $Q_i(x,y)$ are homogeneous polynomials of degree i in x and y with real coefficients. We shall consider the following polynomials:

$$R_i = P_i(x,y)y - Q_i(x,y)x; \quad S_i = \frac{1}{i}\left(\frac{\partial P_i(x,y)}{\partial x} + \frac{\partial Q_i(x,y)}{\partial y}\right), \quad i = 1,\, 4,$$

which in fact are $GL(2,\mathbb{R})$-comitants [1, 2] of the first degree with respect to the coefficients of system (1). Let us consider the following $GL(2,\mathbb{R})$-comitants and $GL(2,\mathbb{R})$-invariants for the system (1), constructed by using the comitants R_i and S_i ($i = 1,4$) and the notion of the transvectant [3] (in the list below, the bracket "$\llbracket$" is used in order to avoid placing the otherwise necessary parenthesis "("):

$$I_1 = S_1, \quad I_2 = (R_1,R_1)^{(2)}, \quad I_3 = \llbracket S_4,R_1)^{(2)},R_1)^{(1)},(S_4,R_1)^{(2)})^{(1)},$$
$$I_4 = \llbracket R_4,R_1)^{(2)},R_1)^{(2)},R_1)^{(1)},((R_4,R_1)^{(2)},R_1)^{(2)})^{(1)}.$$

The system (1) can be written in the following coefficient form:

$$\begin{aligned}\frac{dx}{dt} &= cx + dy + gx^4 + 4hx^3y + 6kx^2y^2 + 4lxy^3 + my^4,\\ \frac{dy}{dt} &= ex + fy + nx^4 + 4px^3y + 6qx^2y^2 + 4rxy^3 + sy^4.\end{aligned} \tag{2}$$

In [4] were established the necessary and sufficient center conditions for the system (1) (or (2)) with $I_1 = 0,\ I_2 > 0,\ I_3 = I_4 = 0$.

In this paper we consider the class of systems (1) (or (2)) with the conditions $I_3 = 0,\ I_1 = 0, I_2 > 0$. The conditions $I_1 = 0, I_2 > 0$ mean that the eigenvalues of the Jacobian matrix at the singular point $(0,0)$ are pure imaginary, i.e., the system has the center or a weak focus at $(0,0)$. The system (2) with $I_1 = 0,\ I_2 > 0$ and $I_3 = 0$ can be reduced by a centeraffine transformation and time scaling to the form:

$$\begin{aligned}\frac{dx}{dt} &= y + gx^4 + 4hx^3y + 6kx^2y^2 + 4lxy^3 + my^4,\\ \frac{dy}{dt} &= -x + nx^4 + 4px^3y - 6hx^2y^2 - 4(g+k+p)xy^3 - ly^4.\end{aligned} \tag{3}$$

In this paper the sufficient center conditions for the origin of coordinates of the phase plane for the bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree with $I_1 = 0,\ I_2 > 0,\ I_3 = 0$ were established.

Theorem. *The system* (3) *has singular point of the center type in the origin of the coordinates of the phase plane of the system if one of the series of the conditions:*

1) $g + p = 0;$
2) $n = h = l = 0;$
3) $n = -2h - 3l, \quad \sqrt{3}g - 16h + 6\sqrt{3}k - 3\sqrt{3}m + 8\sqrt{3}p = 0,$
   $5\sqrt{3}g + 14\sqrt{3}k + 16l + \sqrt{3}m + 8\sqrt{3}p = 0;$
4) $n = -2h - 3l, \quad \sqrt{3}g + 16h + 6\sqrt{3}k - 3\sqrt{3}m + 8\sqrt{3}p = 0,$
   $5\sqrt{3}g + 14\sqrt{3}k - 16l + \sqrt{3}m + 8\sqrt{3}p = 0$

is fulfilled.

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Bibliography

[1] K. S. Sibirsky, *Introduction to the Algebraic Theory of Invariants of Differential Equations.* Manchester University Press, 1988.

[2] N.I. Vulpe, *Polynomial bases of comitants of differential systems and their applications in qualitative theory.* Kishinev, Shtiintsa, 1986, (in Russian).

[3] G. B. Gurevich, *Foundations of the Theory of Algebraic Invariants.* Noordhoff, Groningen, 1964.

[4] Iu. Calin, St. Ciubotaru, *The Lyapunov quantities and the center conditions for a class of bidimensional polynomial systems of differential equations with nonlinearities of the fourth degree.* Buletinul Academiei de Ştiinţe a Republicii Moldova, Chişinău, 2017.