MODELLING OF PROCESS OF HEATING FRYING SURFACES INTERMEDIATE HEAT-AGENT

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1. INTRODUCTION

The working surface of the frying apparatus with intermediate heating agent is heated by convection heat exchange between heating agent and heaters. The problem of convection heat transfer in a closed surface is a complicated task to solve.

2. RECENT INVESTIGATIONS

There is no analytical solution of this problem today. Thus, to model convectional heat transfer empirical the analytical method is generally used. It requires numerous experiments to be carried out in order to determine rational thermal parameter of the object studied.

The purpose and objectives of the work consists in obtaining an approximate analytical solution of the problem of temperature field unevenness of the working surface of the heating apparatus with indirect heating.

3. RESULTS OF RESEARCH

An approximate physical model of heat transfer in oil casing is obtained on the following assumption. We assume that the nature of convection flow is static, flat and symmetrical relative to heaters (Fig. 1), where rising liquid flow from heaters to the frying surface and downward flow from the frying surface to the bottom of the oil casing takes place. So we have a circular flow *ABCD*.

In such a flow, the maximum and minimum temperatures on the frying surfaces will be observed respectively at points *B, C* along axis *x*. This temperature difference *ΔΘ* is caused by heat emission from the intermediate carrier to the frying surface area between points *A, C*. We suppose that practically there is no heat transfer by thermal conduction as compared to convection.

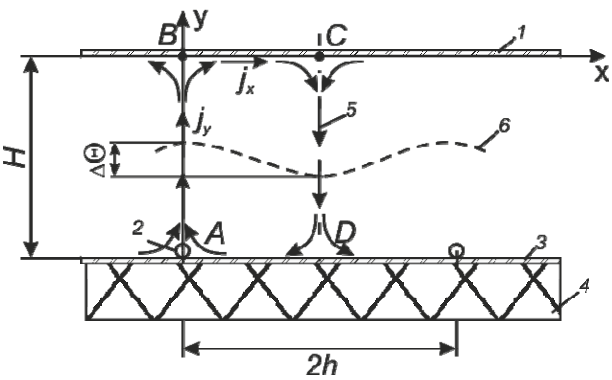

Figure 1. Geometry of convection flows in the oil casing of the multipurpose frying apparatus*: 1 – frying surface; 2 – heaters, 3 – bottom; 4 – casing heat insulation; 5 – convection flows; 6 – character of the temperature field.*

The average velocity convection *v* results from fluid density differences upward and downward flows. Thus Bernoulli equation for viscous fluids can be written this way

$$\frac{\rho v^2}{2} + \Delta\rho g H + \eta \frac{v}{h/2} = 0\,, \qquad (1)$$

where ρ - is the heating agent density for its average temperature, $\Delta\rho$ - is the heating agent density difference in upward and downward flows, v – is the average heating agent velocity, g – is the gravity, H – is the oil casing height, $h/2$ – is a typical circulating current tube size ($2h$ – spacing between heaters), η - is the dynamic heating agent viscosity.

Solution of equation (1) allows to get the hydrodynamic model for free convection (2) in the frying apparatus oil casing with indirect heating if Gr>>1

$$\mathrm{Re} \approx \sqrt{2\,\mathrm{Gr}\frac{H}{h}} \qquad (2)$$

where $\mathrm{Re} = \dfrac{\rho v h}{\eta}$ - is the Reynolds criterion, $\mathrm{Gr} = \dfrac{\rho^2 g h^3 \beta \Delta T}{\eta^2}$ - is the Grashof criterion.

As follows from (2), the circulation velocity depends on maximum temperature difference at points *B, C*. On the other hand, in case of constant frying surface heat loading with higher circulation velocities, circulation losses of circulation flow enthalpy will decrease in area *B, C,* so will decrease temperature difference ΔT. So, to find the temperature distribution on the frying surface along axis *x* the heat exchange problem will be solved on the basis of the following model.

The thermal convection flow change in the infinitely small area Δx due to heat emission to the frying surface makes

$$\Delta j_{x+\Delta x} = \rho c v \frac{dT}{dx}\Delta x\,, \qquad (3)$$

where *c* - is the heating agent thermal capacity,
or

$$\frac{dj_x}{dx} = \rho c v \frac{dT}{dx}\,, \qquad (4)$$

Otherwise, if we neglect heat transfer along axis *x* due to heat conductance, the change of this heat flow must be equal to the flow change along axis *y*, namely

$$\frac{dj_x}{dx} = \frac{dj_y}{dy} = k\frac{d}{dy}(T - T_{жс})\,, \qquad (5)$$

where *k* – is the coefficient of heat transfer from intermediate heating agent to the fat on the apparatus work surface

$$k = \left(\frac{1}{\alpha_m} + \frac{\delta_{жсп}}{\lambda_{жсп}} + \frac{1}{\alpha_{жс}}\right)^{-1}, \qquad (6)$$

where α_m - is the coefficient of heat emission from intermediate heating agent to the work surface, $\delta_{жсп}$ - is the thickness of the frying stove, $\lambda_{жсп}$ - is the coefficient of frying stove heat transfer, $\alpha_{жс}$ - is the coefficient of heat emission from the work surface to the fat.

We use the Newton-Richman equation to find heat transfer between heating agent and frying surface assuming that there is no fluid convection flow towards axis *y* in the area *BC*

$$-\lambda\frac{dT}{dy} = k(T - T_{жс})\,; \qquad (7)$$

where λ - is coefficient of intermediate heat agent thermal conductivity.

Given (4), (5) and (7), we get the form of criteria

$$\mathrm{Re}\frac{d\Theta}{d\xi} = \frac{\mathrm{Nu}^2}{R_m^{\ 2}\,\mathrm{Pr}}(1 - \Theta)\,; \qquad (8)$$

where $\mathrm{Nu} = \dfrac{\alpha_m h}{\lambda}$ - is Nusselt criterion, $R_m = 1 + \mathrm{Bi} + \dfrac{\alpha_m}{\alpha_{жс}}$ - is the relative thermal resistance (heating agent – frying stove – fat), $\mathrm{Bi} = \dfrac{\alpha_m \delta_{жсп}}{\lambda_{жсп}}$ - is Bio criterion, $\mathrm{Pr} = \dfrac{c\eta}{\lambda}$ - is Prandtl criterion, $\Theta = \dfrac{\Delta T}{T_B - T_{жс}}$; $\xi = \dfrac{x}{h}$,

where

$$\Delta T(x) = T_B - T(x); \quad T(x) = T_B - \Delta T(x)\,.$$

We consider that the Reynolds number is associated with dimensionless temperature $\Theta(x)$ in accordance with model (2)

$$\mathrm{Re} = \sqrt{2\,\mathrm{Gr}_0\,\Theta\frac{H}{h}}\,. \qquad (9)$$

where $\mathrm{Gr}_0 = \dfrac{\rho^2 g h^3 \beta (T_B - T_{жс})}{\eta^2}$ - is Grashof criterion for maximum temperature difference between points *B* and *C*.

By substituting equation (9) in (8) we have a differential equation which describes temperature difference caused by thermal emission from the frying surface along axis *x*.

$$\sqrt{2\,\mathrm{Gr}_0\,\Theta\frac{H}{h}}\cdot\frac{d\Theta}{d\xi}=\frac{\mathrm{Nu}^2}{R_m{}^2\,\mathrm{Pr}}(1-\Theta)\,,\quad(10)$$

The solution of this equation with boundary condition $\Theta(0) = 0$ has the form

$$\ln\left(\frac{1+\sqrt{\Theta}}{1-\sqrt{\Theta}}\right)-2\sqrt{\Theta}=\frac{\mathrm{Nu}^2}{R_m{}^2\,\mathrm{Pr}\sqrt{2\,\mathrm{Gr}_0\frac{H}{h}}}\xi\,,\quad(11)$$

If a relative temperature difference is small *(Θ <<1*, the last equation has analytical solution against temperature, where $\Delta\Theta=\Theta(1)-\Theta(0)$ is the relative temperature difference between points of frying surface *C, B*.

The last equation can be written in the form which is appropriate for engineer solutions

$$\Delta\Theta\approx 1{,}04\left(\frac{\mathrm{Nu}}{R_m}\right)^{1{,}33}\mathrm{Pr}^{-0.67}\mathrm{Gr}_0{}^{-0{,}33}\left(\frac{H}{b}\right)^{-0.33}\quad(12)$$

On the outer surface, which comes in contact with food oil, the temperature difference is defined by taking into account the heat flow steadiness

$$\left(T-T_{\text{ж}}\right)\left(\frac{1}{\alpha_m}+\frac{\delta_{\text{жп}}}{\lambda_{\text{жп}}}+\frac{1}{\alpha_{\text{ж}}}\right)^{-1}=\left(T-T_{\text{жп}}\right)\left(\frac{1}{\alpha_m}+\frac{\delta_{\text{жп}}}{\lambda_{\text{жп}}}\right)^{-1}\quad(13)$$

Hence, we have the dimensionless temperature difference on the outer stove surface

$$\Delta\Theta_{\text{жп}}=\Delta\Theta\left(\frac{\dfrac{\alpha_m}{\alpha_{\text{ж}}}}{1+\mathrm{Bi}+\dfrac{\alpha_m}{\alpha_{\text{ж}}}}\right),\quad(14)$$

where $\Delta\Theta_{\text{жп}}=\dfrac{\Delta T_{\text{жп}}}{T_B-T_{\text{ж}}}$ - is the dimensionless temperature difference on the outer surface of the frying stove,

$$\Delta T_{\text{жп}}=T^B_{\text{жп}}-T^C_{\text{жп}}\,,$$

where $T^B_{\text{жп}},T^C_{\text{жп}}$ - are the temperatures at points *B, C* on the outer surface of the frying stove.

Analyzing the last equation we can conclude that for the small thermal resistance (Bi <<1) and large heat transfer coefficient from intermediate carrier to the frying surface ($\alpha_m >> \alpha_{\text{ж}}$) the temperature difference on the outer and inner surfaces of the frying plate are practically identical. For the large thermal resistance (Bi>> $\alpha_m/\alpha_{\text{ж}}$) the temperature difference on the outer surface is much less than on the inner one. Thus, by increasing thickness and decreasing thermal conductivity of the frying stove we improve the uniformity index of the apparatus work surface.

Let's analyze the influence of frying apparatus geometrical and thermophysical parameters on temperature uniformity under indirect heating.

In a general case, the Nusselt number under free convection in the oiling casing is also a function of the heating agent thermophysical properties, casing geometry and spacing of heaters. In the first approximation, we can apply the formula for calculating the Nusselt criterion for free convection in a confined space under the turbulent flow ($\mathrm{Gr}>4\cdot10^5$) between two infinite evenly heated parallel plates [1]

$$\mathrm{Nu}=0{,}068\,\mathrm{Gr}^{1/3}\quad(15)$$

If $\mathrm{Gr}=\mathrm{Gr}_0\cdot\Delta\Theta$, then instead of (13) we have

$$\Delta\Theta\approx 1{,}69\cdot10^{-3}\left(R_m\right)^{-1{,}33}\mathrm{Pr}^{-1.2}\mathrm{Gr}_0{}^{0{,}2}\left(\frac{H}{b}\right)^{-0.6}\quad(16)$$

Formula (16) implies that temperature field unevenness is proportional to the Grashoff - Prandl criteria relationship. Fig.2 shows the estimated frying surface heat unevenness against geometrical simplex *H/h* and thermophysical simplex $\mathrm{Gr}_0/(R_m\cdot\mathrm{Pr})$.

The resulting model of frying surface temperature field unevenness allows to determine thermophysical characteristics of the frying apparatus. So, intermediate heaters should be chosen as the ones which have thermophysical characteristics with minimum simplex value $\mathrm{Gr}_0/(R_m\cdot\mathrm{Pr})$. As for the oiling casing geometry, increase in its height and decrease in the distance between heaters reduce temperature unevenness.

Maximum temperature unevenness should be determined from the following considerations. Culinary product being fried has two characteristic temperatures: the temperature of culinary ripeness in the centre of the product and the maximum available temperature of the surface. Product temperature is a function of frying process duration and surface temperature. Thus, in the places where the temperature of frying surface is minimal, the limiting factor is the

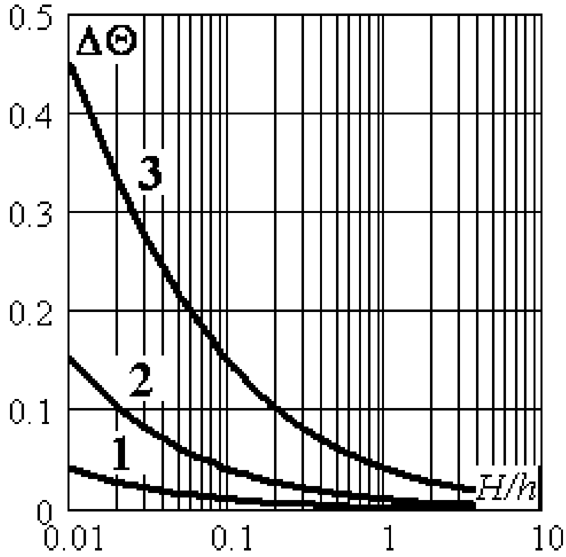

Figure 2. Influence of geometrical and thermophysical factors on the relative temperature ununiformity of the frying surface: 1 – $Gr_0/(R_m \cdot Pr)=10$; 2 – $Gr_0/(R_m \cdot Pr)=10^4$; 3 – $Gr_0/(R_m \cdot Pr)=10^7$.

temperature of culinary readiness, and in the places with maximum temperature, the limiting factor is the maximum permissible temperature of the product surface.

4. CONCLUSIONS

The thermal parameter model of the multipurpose apparatus with intermediate heating which permits to chose geometry of the oil casing and intermediate heating agent based on a minimum temperature difference on the frying surface was obtained.

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